On algebraically integrable domains in Euclidean spaces

Mark Agranovsky (Bar-Ilan Univeristy)

Abstract: Motivated by problems in celestial mechanics, Newton proved that infinitely smooth ovals in the plane are never algebraically integrable, meaning that the area cut off from an oval in the plane by a straight line never depends algebraically on the line. Arnold suggested to find all infinitely smooth algebraically integrable domains in the spaces $\mathbb{R}^n$ of arbitrary dimensions and conjectured that the only such domains are the odd-dimensional ellipsoids. The talk will be devoted to recent progress in proving this conjecture.

Continuous functions in de Branges-Rovnyak spaces

Alexandru Aleman (Lund University)

Abstract: If b is analytic function in the unit disc bounded by 1, the corresponding de Branges-Rovnyak spaces $\mathcal{H}(b)$ is the Hilbert spaces of analytic functions with reproducing kernel $k_w(z) = \frac{1-b(z)\overline{b(w)}}{1-z\overline{w}}$. The structure of these spaces depends essentially on the integrability of $\log(1-|b|)$ on the unit circle. If this function is integrable, $\mathcal{H}(b)$ contains the polynomials as a dense subset. The aim of the talk is to show that even if $\log(1-|b|)$ is not integrable, the functions which belong to the disc algebra and to $\mathcal{H}(b)$ are dense in the space. This is a report about joint work with Bartosz Malman.

Extremal problems in the work of Dima Khavinson

Catherine Beneteau (University of South Florida)

Abstract: In this talk I will give a survey of the many contributions of Dima Khavinson to the world of extremal problems in analytic function spaces. I will pay special attention to certain linear and non-linear problems in Bergman spaces, including the notion of contractive divisor, as well as the concept of analytic content.

On the number of solutions of some transcendental equations

Alexandre Eremenko (Purdue University)

Abstract: A survey of some recent results on the upper estimates of the number of solutions of equations $\overline{z} = f(z)$, where f is an analytic function, will be given. These equations frequently arise in applications, and a method of estimating the number of their solutions was invented by Khavinson, Swiatek and Neumann. New results obtained with this method will be discussed.

Harmonic functions which vanish on a cylindrical surface

Stephen Gardiner (University College Dublin)

Abstract: The Schwarz reflection principle is a beautiful and important result concerning the extension of a harmonic function h on a domain Ω in $\mathbb{R}^N$ through a relatively open subset E of the boundary on which h vanishes. The extension is defined by means of a simple formula, and the domain of extension is independent of the choice of h. When $N = 2$ such a reflection principle holds whenever E is contained in an analytic arc. When $N = 3$, Ebenfelt and Khavinson have shown that a point-to-point reflection law can only hold when the containing real analytic surface is either a hyperplane or a sphere. Thus, for other surfaces, more elaborate arguments are required to investigate whether such harmonic extension is still possible.

An important particular case concerns cylindrical surfaces, since a cylinder is the Cartesian product of a line and a sphere, each of which separately admits Schwarz reflection. Dima Khavinson asked whether a function which is harmonic on a circular cylinder in $\mathbb{R}^3$ and vanishes on the boundary must automatically have a harmonic extension to the whole of space. This talk will present an affirmative answer to this question, and describe several other recent extension results for harmonic functions which vanish on cylindrical surfaces.

This is joint work with Hermann Render.

TBA

Björn Gustafsson (KTH Royal Institute of Technology)

Abstract:

TBA

Håkan Hedenmalm (KTH Royal Institute of Technology)

Abstract:

The Schwarz Potential: an overview of various aspects

Lavi Karp (ORT Braude College of Engineering)

Abstract: The notion of the Schwarz potential as we are all familiar with, was first introduced by Khavinson and Shapiro in the manuscript *The Schwarz potential in* $\mathbb{R}^n$ *and Cauchy's problem for the Laplace equation* in 1989. The paper has evoked a long and significant research of various problems in potential theory attached to partial differential equations. The talk will review divers developments of the various themes of the paper, as well as the progress achieved throughout the years. We will emphasize global aspects of the Cauchy problem, growth properties of the Schwarz potential, behavior of the free boundaries, and construction of quadrature domains in the space by means of a computation of the Schwarz potential.

Clark model for finite rank perturbations

Constanze Liaw (Baylor University)

Abstract: The unitary perturbations of a given unitary operator by finite rank d operators can be parametrized by $d \times d$ unitary matrices; this generalizes the rank $d = 1$ setting, where the Clark family is parametrized by the scalars on the unit circle. For finite rank perturbations we investigate the functional model of a related class of contractions, as well as a (unitary) Clark operator that realizes such a model representation for a particular contraction. We find a universal representation of the adjoint of the Clark operator, which features a matrix-valued Cauchy integral operator. By universal we simply mean that our formula is given in the coordinate free Nikolski–Vasyunin functional model.

We express the matrix-valued characteristic functions of the model (for the class of contractions). In the case of inner characteristic functions results suggest a generalization of the normalized Cauchy transform to the finite rank setting.

This presentation is based on joint work with Sergei Treil.

TBA

Svitlana Mayboroda (University of Minnesota)

Abstract:

Second-order L^2-regularity in nonlinear elliptic problems

Vladimir Maz'ya (Linköping University)

Abstract: A second-order regularity theory is developed for solutions to a class of quasilinear elliptic equations in divergence form, including the p-Laplace equation, with merely square-integrable right-hand side. Our results amount to the existence and square integrability of the weak derivatives of the nonlinear expression of the gradient under the divergence operator. This provides a nonlinear counterpart of the classical L^2-coercivity theory for linear problems, which is missing in the existing literature. Both local and global estimates are established. The latter apply to solutions to either Dirichlet or Neumann boundary value problems. Minimal regularity on the boundary of the domain is required. If the domain is convex, no regularity of its boundary is needed at all. This is a joint work with A. Cianchi.

Reflection principles and harmonic extensions

Hermann Render (University College Dublin)

Abstract: In the first part of this talk a survey about reection principles for elliptic second order PDEs is presented. Special emphasis is given to reflection laws in generalized axially symmetric potential theory which is important for problems with cylindrical geometry.

The second part is devoted to recent joint work with S.J. Gardiner concerning the harmonic extendibility of harmonic functions vanishing on a cylindrical surface. In this approach the existence of a harmonic extension was established by an analysis of the convergence properties of a double series expansion of the Green function of the infinite cylinder or an infinite annular cylinder beyond the domain. The final aim of the talk is to discuss this extension property and to find deeper connections to reflection principles.

Interpolation and complete Pick spaces

Stefan Richter (University of Tennessee)

Abstract: Let X be a set. A Hilbert space $\mathcal{H}_k$ of functions on X is called a complete Pick space, if it has a reproducing kernel of the type

$$k_w(z) = \frac{1}{1 - \sum_n \overline{u_n(w)} u_n(z)}.$$

The Hardy and Dirichlet spaces of the unit disc and the Drury-Arveson space of the unit ball are well-known examples of such spaces.

We prove that a sequence of points in X is interpolating for such a space $\mathcal{H}_k$, if and only if it is separated and satisfies a Carleson condition for the space. This theorem generalizes classical results of Carleson, Shapiro-Shields, Bishop, Marshall-Sundberg, and others. The main new ingredient in the proof of the general result is a use of the Marcus-Spielman-Srivastava Theorem.

Furthermore, we provide a condition on a positive integer N and a Bekolle weight ω in the unit ball $\mathbb{B}_d$ of $\mathbb{C}^d$ that is sufficient for the space

$$B^N_\omega = \{f \in \text{ Hol}(\mathbb{B}_d) : \int_{\mathbb{B}_d} |R^N f|^2 \omega dV < \infty\}$$

to be a complete Pick space. Here $R = \sum_{k=1}^d z_k \frac{\partial}{\partial z_k}$.

This is joint work with Alexandru Aleman, Michael Hartz, and John McCarthy.

Carlson's Theorem for Different Measures

Meredith Sargent (Washington University in St. Louis)

Abstract: We use an observation of Bohr connecting Dirichlet series in the right half plane $\mathbb{C}_+$ to interpret Carlson's theorem about integrals in the mean as a special case of the ergodic theorem by considering any vertical line in the half plane as an ergodic flow on the polytorus. Of particular interest is the imaginary axis because Carlson's theorem for Lebesgue measure does not hold there. We construct measures for which Carlson's theorem does hold on the imaginary axis for functions in the Dirichlet series analog of the disk algebra $\mathcal{A}(\mathbb{C}_+)$.

Orthogonal polynomials and zeros of optimal approximants

Daniel Seco (Universitat de Barcelona)

Abstract: We study connections between orthogonal polynomials, reproducing kernels, and polynomials p minimizing Dirichlet-type norms $\|pf-1\|$ for a given function f (which we call optimal approximants to $1/f$). We exploit them in order to determine the regions in the complex plane that are free from the zeros of the approximants in a large class of Hilbert spaces.

On mother body measures with algebraic Cauchy transform

Boris Shapiro (Stockholm University)

Abstract: We discuss the existence of a mother body measure for the exterior inverse problem in potential theory in the complex plane. More exactly, we study the question of representability almost everywhere (a.e.) in $\mathbb{C}$ of (a branch of) an irreducible algebraic function as the Cauchy transform of a signed measure supported on a finite number of compact semi-analytic curves and a finite number of isolated points. Firstly, we present a large class of algebraic functions for which there (conjecturally) always exists a positive measure with the above properties. This class was discovered in our earlier study of exactly solvable linear differential operators. Secondly, we investigate in detail the representability problem in the case when the Cauchy transform satisfies a quadratic equation with polynomial coefficients a.e. in $\mathbb{C}$.

Fingerprints of Lemniscates via Quadratic Differentials

Alexander Solynin (Texas Tech University)

Abstract: We will discuss several aspects of mathematical theory designed for recognition of two-dimensional images or "shapes". An idea to use the so-called "fingerprints" to study two-dimensional "shapes" goes back to a paper "*Kähler structure on the K -orbits of a group of diffeomorphisms of the circle*" of Alexander Kirillov published in 1987. But it were Eiten Sharon and David Mumford who turned this idea (in their 2004 paper "*2d-shape analysis using conformal mapping*") into a tool, which can be applied for recognition of planar shapes, such as shapes on TV screens and in pictures. After that this theory became quit a popular topic in recent publications on applications of complex analysis to problems in pattern recognition.

An interesting approach to fingerprint problem was suggested by Peter Ebenfelt, Dima Khavinson and Harold Shapiro in their paper "*Two-dimensional shapes and lemniscates*" published in 2011. In this paper, the authors have shown, in particular, that fingerprints of polynomial lemniscates (which, by a classical result due to David Hilbert, are dense in the space of all two-dimensional shapes) are generated by solutions of functional equations, which involve Blaschke products. A simpler proof of the main result of Ebenfelt, Khavinson and Shapiro and its generalization to the case of rational lemniscates were presented in a nice short paper "*Shapes, fingerprints and rational lemniscates*" by Malik Younsi published in 2016.

The first goal of this talk is to discuss how methods of Complex Analysis can be applied to the problems of pattern recognition. In particular, I will discuss the main results on fingerprints obtained by Ebenfelt, Khavinson and Shapiro and by Younsi. In addition, I will also mention a different approach to fingerprints via circle packing which was used by Brock Williams.

My second goal here is to present my recent results, which include as special cases Ebenfelt-Khavinson-Shapiro characterization of fingerprints of polynomial lemniscates as well as Younsi characterization of rational lemniscates. My main intention here is to emphasize the role of *quadratic differentials* in this developing theory.

Stochastic regularization of singularities in free boundary problems

Razvan Teodorescu (University of South Florida)

Abstract: Singularity formation in nonlinear dynamical systems is a broad subject with many implications, especially in the case of fluid dynamics, where it can be seen as an essential component of phenomena such as turbulence, wave-breaking, and fingering instabilities. Often, the mathematical model exhibiting these features is obtained from the original one by a multi-scale averaging procedure, and the only known approach to avoiding singularity formation is to restore the (more complicated, and often unsolved) original problem. In this talk, we discuss another approach, in which a dynamical system approaching a singular point is perturbed stochastically, which allows to avoid the critical point while retaining the characteristics of the original problem. Generalization in the sense of catastrophe theory will also be discussed.

Validated numerical analysis of the Hele-Shaw free boundary problem

Andrew Thomack (Florida Atlantic University)

Abstract: We consider the classical Hele-Shaw free boundary problem of tracking the shape of a two-dimensional region of viscous fluid driven by a source or sink. When surface tension is neglected, the problem reduces, as a consequence of Richardson's Theorem, to solving an inverse moment problem. The problem is further simplified if the conformal map from the unit disk to the initial domain is assumed to take a simplified form, say, a polynomial. The explicit solution to certain low-degree instances are well-known. For more complicated initial domains, we propose an approach using rigorous numerics carried out in the coefficient space of the conformal map.