Building the semiclassical limit of graphs

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Pizza Seminar 4/26/24

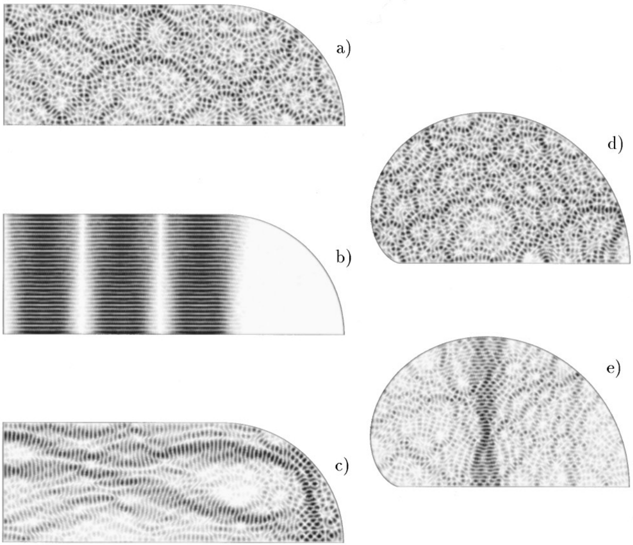


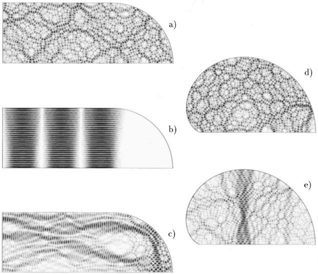
Figure 1: Image Bäcker, Schubert and Stifter

Quantum ergodicity

Theorem 1 (Schnirelman, Zelditch, Colin de Verdière)

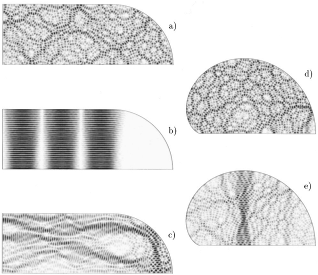
For ergodic dynamics and $\{\psi_{j_k}\}$ a subsequence of eigenfunctions with density one,

$$\int_R |\psi_{j_k}|^2 \, dA \to \frac{\mathrm{Vol}(R)}{\mathrm{Vol}(\Omega)} \, .$$



Quantum Unique Ergodicity (QUE)

$$\int_R |\psi_{j_k}|^2 \, dA \to \frac{\mathrm{Vol}(R)}{\mathrm{Vol}(\Omega)} \text{ for every subsequence } \{\psi_{j_k}\}.$$



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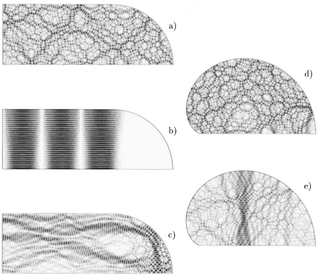

- QUE on arithmetic surfaces, Lindenstrauss '06.
- Compact negatively curved manifolds are at least half delocalized, Anantharaman-Nonnenmacher '07.
- Ergodic billiards are not quantum unique ergodic, Hassell '10.

QUE on Circulant graphs

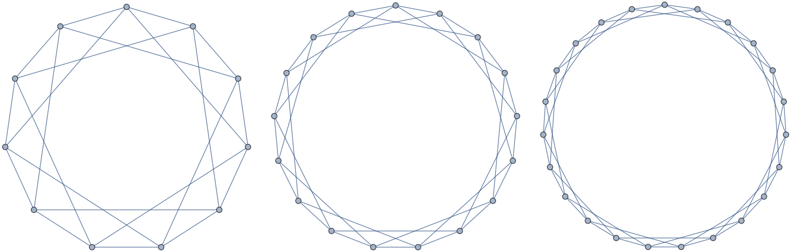

Figure 2: Circulant graphs $C_{11}(1,3)$, $C_{17}(1,3)$ and $C_{23}(1,3)$.