Uncertainty, Contracting, and Beliefs in Organizations*

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Abstract

We study the impact of uncertainty on optimal contracting in a multidivisional firm. Headquarters contracts with division managers to induce effort. Uncertainty creates endogenous disagreement, aggravating moral hazard. By hedging uncertainty, headquarters designs incentive contracts that reduce disagreement and lower incentive provision costs, thereby promoting effort. Because hedging uncertainty can conflict with hedging risk, optimal contracts differ from those in standard principal-agent models. Our model helps explain the prevalence of equity-based incentive contracts and the rarity of relative-performance contracts, especially in firms facing greater uncertainty.

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The provision of incentives in organizations is essential for economic efficiency. A key question is how to determine appropriate performance measures for incentive pay. Managerial contracts often combine base pay, geared to narrowly defined division-specific performance measures ("pay-for-performance"), with a component linked to overall firm profitability (e.g., bonuses, equity-based pay, and other "aggregate" performance measures).[1] This distinction is essential for lower-level managers. The case is strong for using equity-based incentives for top managers who are responsible for the performance of the overall firm. Absent interdependencies across divisions, the use of equity-based pay for division managers and rank-and-file employees is more puzzling. For lower-level employees, equity-based compensation reduces the responsiveness of pay to actions, weakening incentives at the cost of increasing their overall risk exposure. In addition, when cash flows are positively correlated across divisions, incentive contracts should include a relative-performance component to reduce harmful risk bearing, a feature more rarely observed in practice.

We study the impact of uncertainty (or "ambiguity") aversion on incentive contracts in organizations.[2] Our key feature is the acknowledgement that most corporate decisions are taken without full knowledge of the probability distributions involved, a situation characterized as uncertainty (Knight, 1921). We consider a multi-division firm with headquarters (HQ) and (two) division managers. Division cash flows depend on unobservable effort exerted by division managers. Division managers (and HQ) are uncertain on division productivity, which affects their incentives to exert effort. To isolate the effect of uncertainty on incentive pay, in the basic model we rule out synergies or other interdependencies across divisions (as in Holmström, 1982).

Traditional principal-agent theory (Holmström, 1979) suggests that, to limit risk exposure, incentive contracts should depend only on performance measures that are informative on actions (the "informativeness principle," Holmström, 2017).[3] An implication is that incentive contracts should hedge division managers' risk by giving a negative (positive) exposure to variables positively (negatively) correlated to division cash flow. In our setting, HQ can (partially) hedge division

[1] The use of aggregate performance measures, such as bonuses, is documented in the accounting literature (Bushman et al., 1995; Bouwens and Van Lent, 2007; Labro and Omartian, 2022). See Frydman and Jenter (2010), Oyer and Schaefer (2011), Murphy (2013), and Edmans et al. (2017) for extensive surveys.

[2] The importance of ambiguity aversion in affecting individual decision-making has been shown in both experimental and empirical studies (e.g., Epstein and Schneider, 2008; Anderson et al., 2009; Bossaerts et al., 2010; Ju and Miao, 2012; Machina and Siniscalchi, 2014; Jeong et al., 2015; and Hong et al., 2018;).

[3] Responsiveness of CEO pay to risk factors not informative on their actions ("pay-for-luck") has been documented in several studies (e.g., Bertrand and Mullainathan, 2001; Choi et al., 2022).

manager risk exposure by offering contracts with a relative-performance component for a positive correlation of division cash flows or an equity-based component for a negative correlation.

These predictions change substantially in the presence of uncertainty aversion. We model uncertainty aversion by adopting the multiple prior approach of Gilboa and Schmeidler (1989). In this setting, uncertainty-averse agents do not have a single prior but, instead, are endowed with a set of admissible priors (the "core beliefs set") and assess random variables by selecting, from that set, the measure that minimizes their expected utility. We model the core beliefs set based on the relative entropy criterion of Hansen and Sargent (2001, 2008). Intuitively, under the relative entropy criterion, uncertainty-averse agents consider as admissible only probability measures that are not "too unlikely" to be the true distribution given a certain reference probability.

The presence of uncertainty aversion has two adverse effects on incentive provision. First, traditional incentive contracts, by loading primarily on division cash flows, lead uncertainty-averse managers to hold conservative estimates (beliefs) about the productivity of their own division, with a negative impact on their effort. Like Miao and Rivera (2016), we interpret "beliefs" broadly as the probability measure that agents adopt to assess random variables and consequences of their actions. The implication is that HQ must increase pay-for-performance sensitivity to elicit any desired level of effort. Second, uncertainty aversion creates an endogenous disagreement between HQ and division managers on the valuation of incentive contracts, due to each division manager's greater exposure to uncertainty on their own division compared with HQ's exposure to uncertainty (who instead has exposure to the overall firm). The effect of this disagreement is to lead division managers to value compensation contracts at a discount with respect to the value attributed by the more confident HQ. This makes it more difficult to meet their participation constraint, increasing the cost of incentive provision.

The key economic driver in our paper is that uncertainty-averse division managers hold (weakly) more favorable expectations about their own division and, thus, are more confident when incentive pay depends on the performance of both divisions (that is, with cross-pay). The positive effect on beliefs is a consequence of the benefits of uncertainty hedging that stem from the "uncertainty aversion" axiom of Gilboa and Schmeidler (1989).

The benefits of uncertainty hedging under uncertainty aversion are analogous to the traditional benefits of diversification under risk aversion. They may be seen as follows. Pay-for-performance compensation makes uncertainty-averse division managers concerned that the productivity of their own division is very low, depressing their effort incentives. The presence of cross-pay hedges the uncertainty faced by division managers. Consider, for example, an incentive contract with an equity-based pay component. The presence of equity-based pay exposes division managers to uncertainty from *both* divisions, and they will regard the possibility that *both* divisions are characterized by very low productivity sufficiently unlikely to be ruled out by the relative entropy criterion. The effect is to make division managers hold more favorable beliefs about their own division (in fact, about *both* divisions), thereby improving their effort incentives. The implication is that cross-pay (such as equity-based or relative-performance compensation) may be desirable even when division cash flows are uncorrelated, in clear contrast to the informativeness principle.

Our paper offers three main implications. First, we argue that HQ can reduce the negative impact of uncertainty by managing individual exposure to uncertainty through contracts, with beneficial effects on incentives. The role of contracts in managing agents' beliefs through uncertainty hedging is novel in the theory of contract design. It is a direct consequence of the property that beliefs held by uncertainty-averse agents are determined endogenously and depend on their overall exposure to the sources of uncertainty. Differential exposure to uncertainty may be due to the position in the organization (hierarchical exposure) or to the contractual relationships that bind agents (contractual exposure). Hierarchical and contractual exposures concur to determine the prevailing structure of beliefs in an organization. By design of incentive contracts, HQ can affect agents' beliefs with a positive impact on incentives. An implication is that equity-based incentive contracts can be used to realign internal beliefs, which generates consensus by promoting a "shared view" in the organization.[4] The presence of a shared view can reinforce the beneficial effect of equity in fostering internal cooperation.

The second implication is that uncertainty aversion may create a trade-off between hedging risk and hedging uncertainty. When uncertainty faced by division managers is sufficiently large, uncertainty aversion creates the potential for a significant divergence between beliefs held by division managers and those held by HQ. In this case, HQ finds it desirable to hedge division managers'

[4] The role of equity-based compensation to promote consensus in organizations is examined in organization behavior literature, such as Klein (1987), Pearsall et al. (2010), and Blasi et al. (2016), among others. The importance of promoting a shared view is discussed in Zohar and Hofmann (2012). Several papers study the advantages and disadvantages of disagreement: Dessein and Santos (2006); Landier et al. (2009); Bolton et al. (2013); and Van den Steen (2005, 2010).

uncertainty by offering compensation contracts with greater cross-division exposure, even if it comes at the cost of greater risk. The implication is that the presence of uncertainty aversion may lead to incentive contracts that deviate substantially from traditional contracts that hedge only risk.

The third implication is that, in the presence of high uncertainty, it may be desirable to hedge division managers' uncertainty with equity-based rather than relative-performance compensation. This happens when HQ is uncertainty averse as well. HQ uncertainty aversion introduces an additional source of disagreement with division managers, making it costlier to offer incentive contracts based on relative performance. The reason is that relative-performance pay essentially involves division managers and HQ holding opposite positions on the hedging variable (with one party holding a "long" position and the other party a "short" position). Higher uncertainty increases the endogenous disagreement between HQ and division managers, leading them to hold sharply different beliefs. Greater disagreement makes it more expensive for HQ to meet division managers' participation constraint, increasing the cost of incentive provision. The overall effect makes relative-performance contracts costlier and equity-based pay more desirable. Interestingly, we find that equity-based contracts are optimal when uncertainty is sufficiently large, irrespective of the correlation between divisional cash flows. Thus, equity-based compensation can be optimal even when divisional cash flows are positively correlated, in contrast to traditional models. Positive correlation of divisional cash flows is particularly relevant in practice because it may reflect exposure to common aggregate risk factors, such as the business cycle. These features help explain the rarity of relative-performance compensation, especially in firms characterized by high uncertainty, such as young and innovative firms.

The benefits of hedging uncertainty may also be obtained by including appropriate external benchmarks (such as an industry-wide performance index) in executive compensation. Our paper suggests that the use of such benchmarks may be costly. This happens because using such benchmarks can make compensation contracts depend on sources of uncertainty where HQ and employees hold positions of opposite sign. As a consequence, HQ and employees can hold divergent beliefs on the value of compensation, making it more costly for HQ to meet their participation constraints. An important question, therefore, is the selection of specific random variables that are better suited to hedge uncertainty.[5] We show that, with high uncertainty, HQ prefers to hedge uncertainty by

[5] For example, including exposure in incentive contracts to, say, the result of the Super Bowl may provide little or

use of equity-based compensation rather than external hedges.

The benefits of uncertainty hedging in our paper are similar to those discussed in Dicks and Fulghieri (2019, 2021). These papers follow a similar methodology by adopting the approach to uncertainty aversion of Gilboa and Schmeidler (1989) and modeling the core beliefs set using the relative entropy criterion. This paper, however, differs notably from the earlier ones: in our setting, division managers (the "agents") are risk averse (rather than risk neutral as in the earlier papers). Risk aversion is at the heart of traditional principal-agent problems and is a crucial component of our analysis. A key feature of our model is that hedging uncertainty and hedging risk interact in essential ways and may lead to compensation contracts that differ from those that would be optimal under risk aversion and uncertainty aversion in isolation. The tension between hedging risk and hedging uncertainty is a new feature in incentive contract design.

In summary, our paper offers several novel implications that help explain empirical regularities that are difficult to explain based on risk aversion alone. First and foremost, uncertainty aversion can explain the beneficial role of employee bonuses geared to the entire firm's performance (or to that of one of its larger subdivisions) rather than to more narrowly defined performance measures. Second, it can explain the infrequent use of relative-performance compensation and benchmarking despite their well-established benefits within traditional risk aversion.[6] Third, it can explain compensation practices in business groups, whereby compensation depends on the performance of the entire group in addition to the performance of individual units.[7] Finally, our approach provides a framework for belief formation in organizations that can explain the more optimistic attitude toward future firm performance shown by managers higher in the corporate hierarchy relative to rank-and-file employees.[8]

no value in hedging uncertainty relative to its added risk exposure.

[6] Fleckinger (2012) shows that the benefit of relative performance in incentive pay may depend on the impact of effort on the correlation in outcomes. In our paper, correlation is not affected by effort. DeMarzo and Kaniel (2023) argue that relative-performance compensation is not desirable when division managers have "keep-up-with-the-Joneses" preferences.

[7] For example, the compensation of mutual fund managers depends not only on the performance of their funds but also on the performance of the entire family of funds, implying a positive cross-fund exposure (see Ma et al., 2019). However, the majority of funds are exposed to shared macroeconomic risk, suggesting a positive correlation. Similar practices are common in the investment bank industry.

[8] Links between pay and sentiment are shown in several papers, such as Heaton (2002), Oyer and Schaefer (2005), and Bergman and Jenter (2007), among others. In Goel and Thakor (2008), greater optimism of senior management depends on (equilibrium) selection of agents with heterogeneous beliefs. In contrast, in our model, differences in beliefs emerge endogenously among otherwise identical agents as the outcome of differences in their hierarchical and contractual exposure.

Our paper is linked to several streams of literature. The first one is the traditional principal-agent theory and the theory of optimal contract design within organizations, building on the seminal work by Mirrlees (1975), Holmström (1979), and Grossman and Hart (1983). Incentive contracts tailored to shareholder value, such as equity, are shown to be optimal when agents choose their hidden action from rich sets of possible action profiles (Diamond, 1998; Chassang, 2013). Oyer (2004) suggests that equity-based compensation (for example, through stock-option plans) has the advantage of adjusting employees' compensation directly to their outside options (which may be correlated to firm value), facilitating satisfaction of the participation constraint.

The second stream is the emerging literature on contract theory under uncertainty. Lee and Rajan (2020) study the incentive contract between a principal and a single agent where both parties are uncertainty averse but risk neutral, and the source of uncertainty is the probability distribution of the random cash flow. Their paper shows that, contrary to the basic case of uncertainty-neutrality of Innes (1990), the optimal contract has equity-like components. Szydlowski and Yoon (2022) consider a dynamic contracting model where an uncertainty-averse principal designs a dynamic contract for an uncertainty-neutral agent, and the source of uncertainty is the agent's cost of effort. Different from our paper, uncertainty leads principals to increase pay-for-performance sensitivity. Miao and Rivera (2016) consider the optimal contract between an uncertainty-averse principal and an uncertainty- and risk-neutral agent and show that the principal's preference for robustness can cause the incentive-compatibility constraint to be lax.[9] In our paper, we consider multiple risk- and uncertainty-averse agents, creating a new tension between hedging risk and hedging uncertainty through incentive contracts. When agents are both risk- and uncertainty-averse, hedging uncertainty can interact with hedging risk, and the two goals can conflict. When uncertainty is sufficiently large, the uncertainty-hedging motive can overcome the risk-hedging motive, reversing important properties of optimal incentive contracts absent uncertainty concerns.

Closer to our paper, Sung (2022) considers a model in which both principal and agent are uncertain about both the mean and volatility of the technology controlled by the agent. That paper shows that consistent with common practice, optimal incentive contracts include exposure to underlying volatility. Unlike our paper, exposure to uncertain volatility allows principals to design optimal contracts that achieve agreement with the agent. Kellner (2015) examines a principal-

[9] Lee and Rivera (2021) consider optimal liquidity management under ambiguity.

agent model with multiple agents and moral hazard, where the principal is risk and uncertainty neutral; agents can be risk and uncertainty averse; and uncertainty is modeled as smooth ambiguity (Klibanoff et al., 2005), and shows that tournaments are optimal with sufficient uncertainty.

In Carroll (2015), a risk-neutral principal, uncertain about the set of actions available to a risk- and uncertainty-neutral agent, optimally grants the agent a linear contract that aligns their payoffs. Linear (or affine) contracts are optimal robust contracts under very weak assumptions on the source of uncertainty characterizing the set of technologies available to the agent. In the spirit of Holmström (1982), Dai and Toikka (2022) examine a moral hazard in teams problem, where a risk-neutral principal designs contracts robust to uncertainty regarding the underlying game that uncertainty-neutral agents play. Their paper shows that optimal robust contracts must have the property that agents' compensation covaries positively, providing conditions under which optimal robust contracts are linear (or affine). Finally, Walton and Carroll (2022) show that, under mild conditions, optimal contracts are linear within several possible configurations of the organization structure when the principal is risk neutral and agents are risk- and uncertainty-neutral.

1 Uncertainty and contracting

1.1 The basic model

We consider a firm composed of two divisions (or business units) denoted by $d \in \{A, B\}$.[10] Each division is run by a division manager supervised by HQ. At the beginning of the period, $t = 0$, each division manager chooses effort, $a_d \in \mathbb{R}_+$, affecting the probability distribution of their divisional cash flow, Y_d, realized at the end of the period, $t = 1$. We assume that the cash flows $Y \equiv (Y_A, Y_B)$ have a joint normal distribution $N(\mu, \Sigma)$ with mean $\mu \equiv (\mu_A, \mu_B)$ and variance-covariance matrix Σ. Managerial effort affects the means of the distributions, and we set $\mu_d = a_d q_d$, where q_d represents the productivity of division $d \in \{A, B\}$. Division cash flows Y_d are homoskedastic, with variance σ^2, and may be positively or negatively correlated, with correlation coefficient ρ. We assume that effort does not affect the variance-covariance matrix, Σ.[11]

Exerting effort is costly: division managers suffer a pecuniary cost $c_d(a_d)$, where $c_d : \mathbb{R}_+ \to \mathbb{R}_+$ is a continuously differentiable, increasing, and convex function. For tractability, we set $c_d(a_d) =$

[10] Our model can equivalently be interpreted as describing separate divisions of a company, or individual companies in a conglomerate, or separate business units of a "pure-play" firm.

[11] Ball et al. (2020) and Hemmer (2024) study contracts when effort affects Σ.

$\frac{1}{2\theta_d}a_d^2$, where θ_d characterizes efficiency of effort. Division managers have preferences with constant absolute risk aversion (CARA), while HQ is assumed to be risk neutral, for simplicity.[12]

Effort exerted by a division manager is not observable by either HQ or the other division manager, creating moral hazard. To promote effort, HQ offers division managers compensation (i.e., incentive) contracts, $w \equiv \{w_d\}_{d\in\{A,B\}}$. Given compensation contract w_d, a division manager earns an end-of-period payoff $U(w_d) = -e^{-rw_d}$, where r is the coefficient of absolute risk aversion (which we assume to be the same for both division managers).

The game unfolds as follows. At the beginning of the period, $t = 0$, HQ chooses incentive contracts $\{w_d\}_{d\in\{A,B\}}$ for each division manager; HQ can commit to contracts w_d, which are observable to both division managers. After contracts are offered and accepted, division managers simultaneously choose their effort, a_d. At the end of the period, $t = 1$, division managers are compensated according to the realized output Y, and consumption takes place.

1.2 Uncertainty aversion

Contrary to the standard principal-agent paradigm of Holmström (1979), we assume both HQ and division managers are uncertain about the exact probability distribution of the end-of-period cash flows. Specifically, we assume that division managers and HQ are uncertain about division managers' productivity, q_d. The presence of such uncertainty affects the (perceived) marginal productivity of effort and, thus, a division manager's incentive to exert effort.

Following Miao and Rivera (2016) and Dicks and Fulghieri (2019, 2021), we model uncertainty (or "ambiguity") aversion by adopting the minimum expected utility (MEU) approach of Gilboa and Schmeidler (1989) and Chen and Epstein (2002). A key feature of this approach is that agents do not have a single prior on future events but, rather, believe that the probability distribution on future events belongs to a certain set, $\mathcal{P}$, denoted the "core beliefs set," and maximize their minimum expected utility,

$$\mathcal{U} = \min_{p\in\mathcal{P}} E_p\left[U(w)\right], \tag{1}$$

[12] In our setting, division managers and HQ are not subject to limited liability. Limited liability affects in general incentive contract design (see, for example, Sappington, 1983). The results in our paper, however, depend on the benefits of uncertainty hedging (as discussed below), and not the specification of the underlying utility function $U(w)$. We adopt the CARA framework for several important reasons. First, its proven great tractability makes it a common setting in principal-agent models (see, for example, DeMarzo and Kaniel, 2023, and the discussion in Bolton and Dewatripont, 2005, and Edmans and Gabaix, 2016). As such, it provides a clear benchmark to identify the specific role of uncertainty aversion in contract design. Second, and more importantly, because linear contracts, which we consider in this paper, are optimal in a continuous-time principal-agent model with CARA utility (Holmström and Milgrom, 1987).

where p is a probability distribution and U is a von Neumann-Morgenstern utility function. An important implication is that uncertainty-averse agents weakly prefer randomizations over random variables (more precisely, over acts as in Anscombe and Aumann, 1963) rather than each individual variable in isolation. This property is a direct consequence of the uncertainty-aversion axiom of Gilboa and Schmeidler (1989) and is known as "uncertainty hedging."

The benefits of uncertainty hedging are analogous to the traditional benefits of diversification under risk aversion. Intuitively, this feature can be seen by noting that, given two random variables, y_j, $j \in \{1, 2\}$, with joint distribution $p \in \mathcal{P}$, by the minimum operator, we have that

$$\alpha \min_{p\in\mathcal{P}} E_p\left[U\left(y_1\right)\right] + (1-\alpha) \min_{p\in\mathcal{P}} E_p\left[U\left(y_2\right)\right] \leq \min_{p\in\mathcal{P}}\{\alpha E_p\left[U\left(y_1\right)\right] + (1-\alpha) E_p\left[U\left(y_2\right)\right]\} \tag{2}$$

for all $\alpha \in [0, 1]$. The key driver of our paper is that condition (2) can hold as strict inequality.

Following Hansen and Sargent (2001, 2008), we model the core beliefs set $\mathcal{P}$ by using the notion of relative entropy. For a given pair of distributions $\hat{P}(x)$ and $P(x)$, with corresponding densities $\hat{p}(x)$ and $p(x)$, defined on the same probability space, the relative entropy of $\hat{P}(x)$ with respect to $P(x)$ is the Kullback-Leibler divergence of $\hat{P}(x)$ with respect to $P(x)$, namely

$$R\left(\hat{P}(x)|P(x)\right) \equiv \int \hat{p}(x) \ln\left(\frac{\hat{p}(x)}{p(x)}\right) dx. \tag{3}$$

The core beliefs set $\mathcal{P}$ for uncertainty-averse agents is then defined as

$$\mathcal{P}(P(x)) \equiv \{\hat{P} : R\left(\hat{P}(x)|P(x)\right) \leq \eta^P\}. \tag{4}$$

where P represents a given reference probability and $\hat{P}(x)$ is an admissible belief held by the agent. From (3), it is easy to see that the relative entropy of $\hat{P}$ with respect to P represents the (expected) log-likelihood ratio of P, when the "true" probability distribution is $\hat{P}$. The core beliefs set $\mathcal{P}$ can be interpreted as the set of probability distributions $\hat{P}$ that, if true, would not reject the ("null") hypothesis P in a (log) likelihood-ratio test. Intuitively, the relative entropy approach considers as admissible only beliefs that are not "too unlikely" to be the true probability distribution, given the reference probability.[13] The effect is to restrict the core beliefs set by excluding as implausible those probability distributions that give too much weight to extreme events, thereby "trimming" agents' pessimism.

[13] As in Hansen and Sargent (2001, 2008), relative entropy characterizes the extent of "misspecification error" when agents believe that the model is P when the true model is $\hat{P}$.

The distribution P can be interpreted as characterizing an agent's "view" about the true probability $\hat{P}$, where the parameter η^P represents the level of uncertainty aversion characterizing the agent and, thus, the degree of confidence on P. A large value of η^P corresponds to situations where agents are more concerned about uncertainty and, thus, have lower confidence on P. Greater concerns for uncertainty may be due to more uncertainty faced by a decision maker, or a greater aversion to it.

A key feature of the multiple prior approach is that beliefs are endogenous, as determined by the minimum expected utility criterion (1). This implies that agents may hold heterogeneous beliefs due to differences in their overall exposure to uncertainty. In our model, differences in exposure to uncertainty across agents are due to differences in their positions in the organization, "hierarchical exposure," and differences in their compensation contracts, "contractual exposure."

Because in our model agents view as uncertain the productivity q_d of the two divisions, we denote the set of beliefs held by agent $i \in \{HQ, A, B\}$ on division productivity as $K^i(q^i)$, with $\hat{q}^i \equiv (\hat{q}^i_A, \hat{q}^i_B) \in K^i(q^i)$ and $q^i \equiv (q^i_A, q^i_B)$, where $\hat{q}^i_d$ represents the belief held by agent i on the productivity of division d, and q^i_d is the corresponding reference belief. We further assume that division managers and HQ share the same reference probability, and set $q^i_d = q_d$ for all $i \in \{HQ, A, B\}$. This assumption allows us to rule out exogenous differences in beliefs and, rather, to focus on belief heterogeneities arising endogenously from differences in contractual and hierarchical exposures.[14]

A key property of relative entropy, and one that plays the crucial role in our paper, is that the core beliefs set $K^i(q)$, is a strictly convex set with smooth boundaries.[15] This property allows (2) to hold as a strict inequality, making uncertainty hedging valuable. Intuitively, when division managers are exposed only to uncertainty about their own division, they will be concerned about facing the lowest possible level of division productivity, with a (sharply) negative effect on effort. In contrast, when division managers are exposed to uncertainty about both divisions, they will regard the possibility of extreme levels of productivity occurring in both divisions as sufficiently unlikely to be ruled out by the relative entropy criterion, with a beneficial effect on beliefs and thus effort. This implies that, by proper design of incentive contracts, HQ can affect the probability

[14] The effect of exogenous differences in beliefs on investment and financial policy of firms is examined, for example, in Van den Steen (2005, 2010) and Boot and Thakor (2011), among others.

[15] For a general discussion, see Theorems 2.5.3 and 2.7.2 of Cover and Thomas (2006). The main results of our paper depend only on the property that the core beliefs set is strictly convex. This property is shared by core beliefs sets defined by divergences that are strictly monotonic and continuous.

measure (i.e., beliefs) used by division managers to assess the productivity of their division, thereby mitigating the adverse effect of uncertainty on effort.[16]

1.3 The optimal contracting problem

At the beginning of the period, $t = 0$, HQ offers division managers incentive contracts w_d, which may depend on realized output of both divisions, Y. For ease of exposition, we restrict our analysis to the case of linear incentive contracts.[17] Given the linear incentive contracts, we set $w_d(Y) = s_d + \beta_d Y_d + \gamma_d Y_{d'}$, where we can interpret the fixed component, s_d, as a "base pay" and the variable component as the "incentive pay." The incentive pay for division managers may be composed of two parts. The first part is the "pay-for-performance" component, which depends on the realized output of their own division Y_d, and where the coefficient β_d represents the pay-for-performance sensitivity. The second part is a "cross-pay" exposure, which depends on realized output of the other division, $Y_{d'}$; setting $\gamma_d > 0$ represents an equity-based component in compensation, and setting $\gamma_d < 0$ makes compensation depend on the relative performance of the two divisions.

Given the CARA utility, we can write the HQ problem in certainty equivalent form. Given beliefs $\hat{q}^d$ held by a division manager about the productivity of both divisions, and the corresponding beliefs held by HQ, $\hat{q}^{HQ}$, division manager utility function in certainty equivalent form is

$$u_d(\hat{q}^d, a) \equiv E\left[w_d|\hat{q}^d, a\right] - \frac{r}{2}Var(w_d) - c_d\left(a_d\right), \tag{5}$$

where $a \equiv (a_A, a_B)$, and $Var(w_d) = \sigma^2\left(\beta_d^2 + 2\rho\beta_d\gamma_d + \gamma_d^2\right)$ is the variance of incentive pay, w_d. Note that the expected value of incentive pay, $E\left[w_d|\hat{q}^d, a\right]$, depends on division managers' beliefs about the productivity and effort of both their own division (through the pay-for-performance component) and the other division (through the cross-pay component). In contrast, because agents view as uncertain only division productivity, and effort does not affect the variance-covariance matrix Σ, the term $Var(w_d)$ does not depend on a division manager's beliefs and effort.

HQ chooses incentive contracts and action profiles, $\{w_d, a_d\}_{d\in\{A,B\}}$, that solve

$$\max_{\{w,a\}} \quad \min_{\hat{q}^{HQ}\in K^{HQ}} \pi(\hat{q}^{HQ}) \equiv \sum_{d\in\{A,B\}} E\left[Y_d(a_d) - w_d|\hat{q}^{HQ}\right], \tag{6}$$

[16] The property of smooth boundary is violated by rectangular core beliefs sets and is discussed, for example, in Chen and Epstein (2002); we will consider the limiting case of rectangular beliefs in Section 5.2.

[17] In the spirit of Holmström and Milgrom (1987), in an earlier version of this paper, Dicks and Fulghieri (2020) show that in a dynamic, stochastic, continuous-time version of this model with IID ambiguity, as in Chen and Epstein (2002), the solution to the dynamic model is characterized by the solution of a corresponding static problem where HQ offers only linear contracts that depend only on end-of-period cash flows.

subject to the division managers' incentive and participation constraints

$$\max_{a_d} \min_{\hat{q}^d \in K^d} u_d(\hat{q}^d, a) \equiv E\left[w_d | \hat{q}^d, a_d, a_{d'}\right] - \frac{r}{2} Var(w_d) - c_d(a_d), \tag{7}$$

$$\min_{\hat{q}^d \in K^d} u_d(\hat{q}^d, a_d, a_{d'}) \geq u_0 = 0 \tag{8}$$

for $d, d' \in \{A, B\}$, where u_0 is a reservation utility (normalized to zero).[18]

Importantly, note that in problems (7)-(8) a division manager's exposure to uncertainty is endogenous and is determined by the incentive contract, w_d, offered by HQ. This contractual exposure to uncertainty determines a division manager's beliefs, $\hat{q}^d$. In contrast, given the higher-level position in the firm hierarchy, from (6) HQ exposure to uncertainty is determined by its residual claim in the overall firm cash flow, given the incentive contracts offered to both division managers in the firm.[19] HQ hierarchical exposure determines HQ beliefs, $\hat{q}^{HQ}$. The triplet $\{\hat{q}^{HQ}, \hat{q}^A, \hat{q}^B\}$ determines the belief system prevalent in the firm.

Definition 1 *An equilibrium is a set of contracts, $w = \{w_d\}_{d \in \{A,B\}}$, an action profile $\{a_A, a_B\}$, and a belief system $\{\hat{q}^{HQ}, \hat{q}^A, \hat{q}^B\}$, such that:*

(i) Given contracts w and effort, a_d, beliefs $\{\hat{q}^{HQ}, \hat{q}^A, \hat{q}^B\}$ satisfy the worst-case scenario. Formally, $\hat{q}^{HQ}$ solves the inner problem of (6) and $\hat{q}^d$ solves the inner problem of (7);

(ii) Given incentive contracts w, each division manager selects effort, a_d, optimally, solving (7), given the other division manager's action, $a_{d'}$ for $d' \neq d$;

(iii) HQ offer contracts w that maximizes (6) subject to (7)-(8).

The main trade-offs faced by HQ in problem (6)-(8) can be decomposed as follows. Because of the translation invariance of CARA, the fixed component of incentive contracts, s_d, is set to make the participation constraint (8) bind, giving

$$s_d = c_d(a_d) + \frac{r}{2} Var(w_d) - E\left[w_d | \hat{q}^d, a\right]. \tag{9}$$

After substitution into the objective function (6), we obtain

[18] Because HQ's objective function is concave in $\{w, a\}$ and K is convex, Sion's Minimax Theorem applies and the max and min operators in (6) can be switched into a min-max (see Sion, 1958). However, the maximin problem is more intuitive than the corresponding minimax problem. This is because, for uncertainty to be economically relevant, economic agents do not observe the state of the world $\hat{q}^d$ before choosing their effort level (as required in the min-max formulation). Rather, division managers are worried about the state of the world, and choose their action (effort) to maximize their expected utility under the measure that minimizes their utility (that is the "worst case"), given the action they take.

[19] We assume HQ is the full residual claimant in firm cash flow. More generally, HQ could act within the context of incentive contracts issued by a compensation committee, exposing it to contractual exposure as well.

$$\pi = \sum_{d\in\{A,B\}} \left[E(Y_d(a_d)|\hat{q}_d^{HQ}) - \frac{r}{2} Var(w_d) - c_d\left(a_d\right) - \left(E\left[w_d|\hat{q}_d^d, a\right] - E[w_d|\hat{q}_d^{HQ}, a]\right)\right]. \qquad (10)$$

From (10), HQ payoff consists of four components. The first one is the expected value assessed by HQ for the two divisions (given beliefs, $\hat{q}_d^{HQ}$) and division managers' effort levels, a; the second one is given by the required risk premia for division managers, $\frac{r}{2}Var(w_d)$; the third one is the cost of providing effort by division managers, $c_d\left(a_d\right)$. These components are common to the traditional problem without uncertainty. The fourth component, due to uncertainty aversion, is new and is discussed below.

Uncertainty aversion affects incentive contracts through two distinct channels. First, from the incentive constraint (7), effort by division managers, a_d, depends on their "worst-case" scenario, $\hat{q}^d$, negatively affecting effort. This implies that HQ must increase the pay-for-performance sensitivity, β_d, to elicit any desired level of effort, increasing the cost of incentive provision. The worst-case scenario, $\hat{q}^d$, however, is itself endogenous, and depends on a division manager's overall exposure to uncertainty through incentive contract, w_d. The key feature of our paper is that, by hedging uncertainty through incentive contract, HQ can improve division manager assessment of division productivity, $\hat{q}_d^d$, promoting effort. We refer to this channel as the "*incentive effect.*"

The second channel is the divergence between HQ and division managers on the valuation of compensation contracts, as captured by the last term in (10), $E\left[w_d|\hat{q}_d^d, a\right] - E[w_d|\hat{q}_d^{HQ}, a]$. This term acts through division managers' participation constraints (8), and reflects the fact that HQ values compensation contracts at its own worst-case scenario, $\hat{q}^{HQ}$, while division managers value contracts at theirs, $\hat{q}^d$, creating a disagreement on the assessment of the value of an incentive contract to division managers and its cost to HQ. In particular, if HQ is more confident than division managers about division productivity, $\hat{q}_d^{HQ} > \hat{q}_d^d$, then division managers value their compensation contracts at a discount relative to HQ valuation, making it more costly (from HQ's point of view) to satisfy their participation constraints (8). We denote this additional cost of incentive-based pay as an "*uncertainty discount effect.*"

Finally, for tractability and to generate closed-form solutions, similar to Dicks and Fulghieri (2019, 2021) we consider a parametric approximation of the core beliefs set (4).[20] Specifically, we

[20] Tractable closed-form solutions would be possible with the relative entropy criterion only for the case of uncertainty-neutral HQ.

assume that HQ and division managers consider beliefs $\hat{q}^i$ in the neighborhood of the reference probability implied by the pair $q = (q_A, q_B)$, as follows. Define $\chi_d^i = \left|\frac{\hat{q}_d^i - q_d}{q_d}\right|$ as the relative error of player i about division d and the distance measure $D\left(\chi_d^i\right) = -\log\left(1 - \chi_d^i\right)$. We denote the core beliefs set for agent i as

$$K^i(q) \equiv \left\{\hat{q}^i | D\left(\chi_A^i\right) + D\left(\chi_B^i\right) \leq \eta^i\right\}, \tag{11}$$

where η^i characterizes the level of uncertainty faced by agent $i \in \{HQ, A, B\}$.[21] The set described in (11) is plotted in Figure 1 on page 42, with relative entropy for comparison. To obtain closed form solutions, we will at times assume that divisions are symmetric:

$$(S): \theta_A = \theta_B \equiv \theta,\ q_A = q_B \equiv q,\ \eta^A = \eta^B \equiv \eta. \tag{12}$$

2 The no-uncertainty benchmark

As a benchmark, we first characterize the solution to the optimal contracting problem without uncertainty, a setting similar to Holmström and Milgrom (1987). Absent uncertainty concerns, $K^{HQ} = K^d = \{q\}$ and division managers share the same beliefs as HQ.

Lemma 1 *(Holmström and Milgrom) Let HQ be risk neutral: optimal contracts are linear in the end-of-period cash flows of both divisions:* $w_d(h_1) = s_d + \beta_d^* Y_{d,1} + \gamma_d^* Y_{d',1}$*, with*

$$\beta_d^* = \frac{1}{1 + r\sigma^2 (1-\rho^2) / \left(\theta_d q_d^2\right)}, \text{ and } \gamma_d^* = -\rho\beta_d^*, \tag{13}$$

and induce optimal effort $a_d^* = \beta_d^* \theta_d q_d$*, for* $d \in \{A, B\}$*. Furthermore, if condition (S) holds,* $\beta_d^* + \gamma_d^* < 1$ *for* $r\sigma^2 > -\rho\theta q^2/(1-\rho^2)$*.*

For future comparisons, note that under risk neutrality, $r = 0$, the optimal contract makes division managers residual claimants, $\beta_d = 1$, leading to first-best effort.[22] The presence of risk aversion increases the cost of incentive provision and reduces pay-for-performance sensitivity, due to term $r\sigma^2\left(1-\rho^2\right)/\left(\theta_d q_d^2\right)$. If cash flows are correlated, then optimal incentive contracts hedge division managers' risk exposure and depend on the correlation of end-of-period cash flows of both divisions. With positive correlation, HQ sets $\gamma_d < 0$ and contracts display relative-performance compensation; with negative correlation, HQ sets $\gamma_d > 0$ and incentive contracts display an equity component

[21] Note that this characterization of the core beliefs set allows a great degree of tractability: when an economic agent has sufficient positive exposure to both divisions, so that $\hat{q}_d^i < q_d$, the minimization problem is isomorphic to the cost minimization problem with Cobb-Douglass utility. Further, the set is symmetric around $q = (q_A, q_B)$, making uncertainty hedging neutral with respect to positive or negative exposure to cross-division uncertainty.

[22] In this case cross-pay γ_d is indeterminate because hedging risk is irrelevant for risk-neutral agents.

through cross-pay. Hedging division manager risk exposure reduces the cost of incentive provision and allows HQ to increase pay-for-performance sensitivity, improving incentives.[23] When cash flows are uncorrelated, cross-pay generates only incremental risk exposure with no risk-hedging benefit, and optimal contracts set $\gamma_d = 0$ (the "informativeness principle"). Finally, HQ hold

Uncertainty aversion affects incentive contracts in important ways. We start with the simpler case where HQ is uncertainty neutral. We then examine the more realistic (and interesting) case where HQ is uncertainty averse as well. This approach allows us to identify the specific impact of uncertainty aversion by division managers and HQ on optimal incentive contracts.

3 Uncertainty-neutral principal

When HQ is uncertainty neutral, $\hat{q}_d^{HQ} = q_d$ and problem (6)-(8) becomes

$$\max_{\{w_d, a_d\}_{d\in\{A,B\}}} \pi = \sum_{d\in\{A,B\}} E\left[Y_d(a_d) - w_d\left(Y\right)|q_d\right] \tag{14}$$

subject to the incentive and participation constraints

$$\max_{a_d} \min_{\hat{q}_d \in K^d} u_d = E\left[w_d|\hat{q}^d, a_d, a_{d'}\right] - \frac{r}{2}Var(w_d) - c_d\left(a_d\right), \tag{15}$$

$$\min_{\hat{q}_d \in K^d} u_d = E\left[w_d|\hat{q}^d, a_d, a_{d'}\right] - \frac{r}{2}Var(w_d) - c_d\left(a_d\right) \geq 0. \tag{16}$$

We solve problem (14)-(16) in three steps. We first examine the impact of incentive contracts on division manager beliefs, $\hat{q}_d$; next, we examine the impact of incentive contracts on effort, a_d. With these preliminary results, we then characterize the optimal contract solving (14)-(16).

3.1 Incentive contracts and beliefs

We start with the characterization of division managers' belief assessments on the productivity of both divisions, which depend on the pair of incentive contracts offered by HQ. From (5) and (11), given incentive contracts $\{w_d\}_{d\in\{A,B\}}$, division managers' beliefs $\hat{q}^d$ solve

[23] Note that division manager compensation has a pay-for-performance component, β_d^*, and a risk-hedging component, γ_d^*. The risk-hedging component effectively represents a "side bet" between a division manager and HQ. If division cash flows are negatively correlated, $\gamma_d^* > 0$, so that the division manager holds a long position and HQ takes the corresponding short position, raising the possibility that HQ holds a short overall position, with $1 - \beta_d^* - \gamma_d^* < 0$. Because $\gamma_d^* = -\rho\beta_d^*$, the size of the hedging component depends on the risk exposure due to the pay-for-performance component. Increasing the coefficient of risk aversion r lowers β_d^*, reducing the corresponding short position held by HQ. Under condition (S), this implies that the inequality $\beta_d^* + \gamma_d^* < 1$ holds if and only if $r > -\rho\theta q^2/(1-\rho^2)\sigma^2$, that is if the agent is sufficiently risk averse.

$$\min_{\hat{q}^d} \ u_d(\hat{q}^d) \ = \ E\left[w_d|\hat{q}^d, a\right] - \frac{r}{2}Var(w_d) - c_d\left(a_d\right), \tag{17}$$

$$s.t. \qquad \ln\left(\frac{1}{1-\left|\frac{\hat{q}_A^d - q_A}{q_A}\right|}\right) + \ln\left(\frac{1}{1-\left|\frac{\hat{q}_B^d - q_B}{q_B}\right|}\right) \leq \eta^d. \tag{18}$$

Note that incentive contracts offered by HQ must have $\beta_d > 0$, so that division managers will exert strictly positive effort, $a_d > 0$.

Lemma 2 *(Incentive contracts and beliefs) Let $\beta_d a_d > 0$ and $H_d \equiv \frac{|\gamma_d| a_{d'} q_{d'}}{\beta_d a_d q_d}$. A division manager's assessment of division productivity, $\{\hat{q}_d^d, \hat{q}_{d'}^d\}$, depends on contractual exposure, w_d, with*

(i) $\hat{q}_d^d = e^{-\eta^d} q_d$ *for* $H_d \in \left[0, e^{-\eta^d}\right]$

(ii) $\hat{q}_d^d = \left(e^{-\eta^d} H_d\right)^{\frac{1}{2}} q_d$ *for* $H_d \in \left(e^{-\eta^d}, e^{\eta^d}\right)$

(iii) $\hat{q}_d^d = q_d$ *for* $H_d \geq e^{\eta^d}$,

where $\hat{q}_d^d$ is weakly increasing in H_d. Furthermore, $\hat{q}_{d'}^d \gtrless q_{d'}^d$ as $\gamma_d \lesseqgtr 0$.

Division managers' beliefs toward division productivity depend on the relative exposure to the cash flow from each division, measured by H_d. Because H_d affects the relative exposure to uncertainty of the two divisions, we refer to this ratio as the "uncertainty-hedging ratio." In particular, compensation contracts setting $H_d = 1$ equates (i.e., "hedges"), a division manager's exposure to each division uncertainty. The uncertainty-hedging ratio H_d is affected by incentive contract, w_d, and is an increasing function of the cross-division exposure, $|\gamma_d|$.

Several features emerge from Lemma 2 and are illustrated in Figure 1. When HQ grants pay-for-performance only (so that $\gamma_d = 0 = H_d$) or offers incentive contracts with a small exposure to the other division cash flow (leading to a small uncertainty-hedging ratio H_d), as in case (i), division managers assess the prospects of their own division very conservatively, with $\hat{q}_d^d = e^{-\eta} q_d$, disincentivizing effort. This case corresponds to point A in Figure 1.

Division manager assessments of the productivity of their own division, $\hat{q}_d^d$, is an increasing function of their exposure to the other division, $|\gamma_d|$, and of the hedging ratio, H_d. Incentive contracts that offer greater exposure to the other division, as in case (ii), induce division managers to become more confident about their own division, $\hat{q}_d^d$. This case corresponds to points B and B' in Figure 1. Finally, when incentive contracts offer significant exposure to the other division with an even larger value of $|\gamma_d|$, as in case (iii), division managers will become very confident about their own division, setting $\hat{q}_d^d = q_d$. This case corresponds to points C and C' in Figure 1. The beneficial effect on division manager beliefs depends on the absolute value of the cross-division exposure, $|\gamma_d|$, and can be obtained with either an equity-based pay, $\gamma_d > 0$, or relative performance-based pay, $\gamma_d < 0$.

The impact of incentive contracts on a division manager's assessment of the other division's productivity depends on the sign of γ_d. An incentive contract with an equity component, $\gamma_d > 0$, makes division managers more pessimistic about the other division's productivity, $\hat{q}^d_{d'} < q_{d'}$ (this case corresponds to point B in Figure 1). This reflects the property that, when $\gamma_d > 0$, a worse performance in the other division reduces a division manager's compensation, resulting in a more pessimistic assessment of that division's productivity. Similarly, incentive contracts with a relative-performance component, $\gamma_d < 0$, make division managers more optimistic about the other division's productivity, $\hat{q}^d_{d'} > q_{d'}$ (this case corresponds to point B' in Figure 1). The more optimistic assessment reflects the fact that, when $\gamma_d < 0$, better performance in the other division reduces compensation, a reason for division manager concern.

3.2 Incentive contracts and effort

Given beliefs characterized in Lemma 2, a division manager's effort is determined by solving

$$\max_{a_d} \; u_d(a, \hat{q}^d(a, w)) = E\left[w_d | \hat{q}^d, a_d, a_{d'}\right] - \frac{r}{2} Var(w_d) - c_d\left(a_d\right). \tag{19}$$

The Nash equilibrium of effort choice by division managers is characterized as follows.

Lemma 3 *(Uncertainty and effort provision) Given incentive contracts, $\{w_d\}_{d \in \{A,B\}}$, there is a unique Nash equilibrium effort, $\{a_A, a_B\}$ where $a_d = \beta_d \theta_d \hat{q}^d_d$ and division manager beliefs, $\hat{q}^d_d$, are as in Lemma 2. Equilibrium effort a_d is increasing in pay-performance sensitivity, β_d, exposure to the other division, $|\gamma_d|$, efficiency of effort, θ_d, and decreasing in uncertainty, η^d. Further, if $H_d \in \left(e^{-\eta^d}, e^{\eta^d}\right)$, a_d is also increasing in $\beta_{d'}$, $|\gamma_{d'}|$, and $\theta_{d'}$, and decreasing in $\eta^{d'}$.*

If division managers are uncertainty neutral, their optimal level of effort is determined by their own division-based pay, β_d, and is affected by neither their cross-division pay, γ_d, nor the action of the other division manager, $a_{d'}$. The only effect of cross-division exposure is to hedge risk exposure, reducing the cost of incentive provision.

Under uncertainty, incentive contracts affect division manager effort through two distinct channels. The first channel is the traditional direct effect due to pay-for-performance compensation captured by β_d. The second channel is indirect, and it acts through the impact of incentive contracts on division managers' assessment of the productivity of their own division, $\hat{q}_d^d(w_d)$. Specifically, the presence of cross-pay, $|\gamma_d| \neq 0$, reduces the relative exposure of division managers to uncertainty about their own division, increasing the hedging ratio H_d. From Lemma 2, this implies that HQ can use incentive contracts to lead uncertainty-averse managers to hold a more favorable assessment of the productivity of their own division, with positive effect on effort. This channel due to uncertainty hedging is new, and is the key driver of our paper.

Finally, note that uncertainty aversion introduces a strategic complementarity across division managers' levels of effort. From Lemma 2, cross-division exposure, $|\gamma_d| > 0$, makes effort exerted by a division manager, a_d, increasing in effort exerted by the other division manager, $a_{d'}$. Greater effort from the other manager decreases the relative exposure to uncertainty about the division manager's own division, leading to more favorable beliefs and greater effort. This new source of externality is due to uncertainty hedging, and is driven solely by beliefs.[24]

3.3 Uncertainty and incentive contracts

To separate the effect of uncertainty aversion and risk aversion, we consider first the case in which division managers are uncertainty averse but risk neutral. This approach allows us to identify the specific role of uncertainty aversion in contract design and its interaction with risk aversion.

Theorem 1 *(Cross-division exposure and uncertainty hedging) If HQ is both risk and uncertainty neutral and division managers are uncertainty averse but risk neutral, optimal incentive contracts have* $H_d = 1$, *inducing division managers' beliefs to be* $\hat{q}_d^d < q_d$, *and* $\hat{q}_{d'}^d < q_{d'}$ *for* $\gamma > 0$, *and* $\hat{q}_{d'}^d > q_{d'} > \hat{q}_d^d$ *for* $\gamma < 0$. *Optimal contracts set*

$$\beta_d = \frac{1}{1 + 3\left(1 - \hat{q}_d^d / q_d\right)} < 1, \text{ and } |\gamma_d| = \xi_d \beta_d, \tag{20}$$

where $\xi_d \equiv \frac{a_d q_d}{a_{d'} q_{d'}}$. *Pay-for-performance sensitivity,* β_d, *and effort,* a_d, *are both decreasing in uncertainty,* η^d. *If condition (S) holds, then equity is optimal,* $\beta_d = \gamma_d$, *with* $\hat{q}_d^d = \hat{q}_{d'}^d = e^{-\frac{\eta}{2}} q < q$ *and* $\beta_d + \gamma_{d'} < 1$ *for* $\eta > 2 \ln \frac{3}{2}$.

If division managers are uncertainty averse but risk neutral, hedging risk is not a concern. The presence of uncertainty has two adverse effects. First, it lowers division managers' assessment of their productivity, with a detrimental effect on effort (the "incentive effect"). This implies that

[24] Note that the presence of a positive externality across effort choices means that division managers would benefit from coordination. However, the benefits of uncertainty hedging will still be present, even if division managers could coordinate, a possibility that we therefore exclude.

HQ must increase pay-for-performance sensitivity to elicit any desired level of effort. Second, more conservative beliefs reduce the value of incentive contracts as assessed by division managers, relative to the value assessed by the more confident HQ, making it more expensive for HQ to meet their participation constraints (the "uncertainty discount" effect).

The combined effect is to make it costlier for HQ to induce effort, leading to a reduction of the pay-for-performance sensitivity β_d, as captured by the term $3\left(1-\hat{q}_d^d/q_d\right)$ in (20). In addition, greater uncertainty, by increasing the cost of inducing managerial effort, makes pay-for-performance sensitivity β_d, and thus effort a_d, decreasing functions of the disagreement between a division manager and HQ, represented by the term $\hat{q}_d^d/q_d$ in (20).

The role of cross-division exposure, $|\gamma_d|$, is to improve division managers' beliefs by hedging their uncertainty. From Lemma 2, an increase of cross-division exposure (partially) offsets the negative effect of uncertainty on beliefs, promoting effort. Absent risk-aversion considerations, optimal contracts hedge a division manager's exposure to uncertainty by equalizing exposure to cash flow uncertainty from each division, setting $H_d = 1$. Because of uncertainty neutrality, HQ is indifferent between granting compensation based on cross-pay, $\gamma_d > 0$, or on relative performance, $\gamma_d < 0$, as the optimal contracts depends only on the size of the cross-division exposure, $|\gamma_d|$, and not on its sign. The extent of cross-division exposure, $|\gamma_d|$, is still proportional to the pay-for-performance sensitivity parameter, with $|\gamma_d| = \xi_d\beta_d$, where ξ_d represents the hedging factor.[25]

If divisions are symmetric (so that $a_{d'}q_{d'} = a_d q_d$), then the uncertainty hedge ratio, H_d, can be set to unity by the use of pure equity contracts: $\beta = \gamma < 1$. Interestingly, in this case, division managers hold the same beliefs about their own as well as the other division, $\hat{q}_d^d = \hat{q}_{d'}^d = e^{-\frac{\eta}{2}}q$, leading to consensus (that is, a "shared view") in the organization. Also, HQ holds more optimistic beliefs than division managers, $q > \hat{q}_d^d = e^{-\frac{\eta}{2}}q$, making HQ appear as "visionary" in the organization. Note also that, absent risk aversion, a contract with extreme relative performance, with $\gamma = -\beta$, is also optimal. In this case, from Lemma 2, we have $\hat{q}_d^d < q < \hat{q}_{d'}^d$, and division managers are more confident about the other division than they are about their own. This belief configuration creates envy and discord in the organization, a potentially undesirable configuration of internal beliefs caused by relative-performance compensation. In Section 4, we will show that it

[25] In the appendix, we show that the hedging factor ξ_d depends on the relative exposure to uncertainty of the two division managers, η^d, and that cross-division exposure is greater for the (relatively) less confident division managers and for larger divisions.

is never optimal when HQ is uncertainty averse as well.

An important implication of Theorem 1 is that the optimal contract (20) differs in two important ways from the corresponding case of risk-neutral division managers with no uncertainty as shown in Lemma 1 (where division managers become full residual claimants in their own division, with $\beta_d = 1$, and with no role for cross-pay γ_d). First, with uncertainty concerns, making division managers full residual claimants exacerbates pessimism toward their own division, depressing effort. In this case, HQ finds it optimal to reduce pay-for-performance sensitivity, $\beta_d < 1$, and to hedge division manager uncertainty by offering exposure to the other division's uncertainty, setting $|\gamma_d| > 0$. Second, optimal contracts offer cross-pay, $|\gamma_d| \neq 0$, even when division cash flows are uncorrelated, in violation to the informativeness principle. The question is whether this feature holds also in the case of risk-averse division managers, a case we consider next.

3.4 Risk and uncertainty hedging

The presence of risk aversion affects optimal contracts because hedging uncertainty creates a risk exposure, which is costly for risk-averse division managers. Optimal contracts must trade off the relative benefits of risk and uncertainty hedging.

Consider, for simplicity, the case where HQ wishes to implement interior beliefs, as in case (ii) of Lemma 2.[26] The composition of pay-for-performance and cross-division exposure will now depend on the relative size of the two divisions, which affects division managers' uncertainty exposure.

Theorem 2 *(Risk and uncertainty hedging) Let the optimal contract $\{\beta_d, \gamma_d\}$ be such that division managers have interior beliefs, $H_d \in \left(0, e^{\eta^d}\right)$. Then $\gamma_d \neq 0$ and*

$$\beta_d a_d q_d + r\sigma^2 \beta_d^2 = |\gamma_d|\, a_{d'} q_{d'} + r\sigma^2 \gamma_d^2, \tag{21}$$

with $|\gamma_{d'}| > \xi_{d'} \beta_{d'}$ and $|\gamma_d| < \xi_d \beta_d$, for $a_d q_d > a_{d'} q_{d'}$.

With risk-neutral division managers, optimal contracts fully hedge uncertainty and set the uncertainty-hedging ratio to $H_d = 1$, which implies that $\beta_d a_d q_d = |\gamma_d|\, a_{d'} q_{d'}$. With risk-averse division managers, hedging uncertainty through cross-pay is costly because of the risk exposure it creates. In this case HQ optimally induces interior beliefs (as in case (ii) of Lemma 2); from Equation (21), optimal contracts must have $\gamma_d \neq 0$ irrespective of the correlation coefficient. This means that

[26] Focusing on "interior" beliefs in Lemma 2 allows us to equate marginal cost and marginal benefits of both β_d and γ_d. If HQ chooses to implement corner beliefs, the optimal incentive contract will mimic that in Lemma 1 (as shown in the proof of Corollary 1).

optimal contacts include cross-division exposure even when division managers are risk averse and division cash flows are not correlated, in clear contrast with the "informativeness principle."

Condition (21) in Theorem 2 shows that optimal contracts equate the total (expected) cost to HQ of a division manager's contractual compensation from exposure to each division. This cost is the sum of two components: for their own division, it is the sum of the (expected) pay-for-performance component, $\beta_d a_d q_d$, and the corresponding risk premium, $r\sigma^2\beta_d^2$; for the other division, it is the sum of expected cross-pay, $|\gamma_d|\, a_{d'} q_{d'}$, and the corresponding risk premium, $r\sigma^2\gamma_d^2$. With respect to the optimal contract in Theorem 1, the presence of risk aversion has the effect of increasing cross-division exposure for the relatively smaller division, $|\gamma_{d'}| > \beta_{d'}\xi_{d'}$ and of decreasing such exposure for the larger division, $|\gamma_d| < \xi_d\beta_d$.[27]

Corollary 1 *(Uncertainty and cross-pay) Let condition (S) hold. There is a threshold $\bar{\eta}(r,\rho)$ (defined in appendix), with $\bar{\eta}(0,\rho) = 0$, such that:*

(i) If $\eta \leq \bar{\eta}$, optimal contracts have $\beta < \beta^*$ and $\gamma = -\rho\beta$, and induce division managers' beliefs $\hat{q}_d^d = e^{-\eta} q$ and $\hat{q}_{d'}^d = q$. Pay-for-performance sensitivity, β, and Nash equilibrium effort, a, are both decreasing in uncertainty, η; the threshold $\bar{\eta}(r,\rho)$ is increasing in r and $|\rho|$.

(ii) If $\eta > \bar{\eta}$, optimal contracts have $|\gamma| = \beta < \beta^*$, with $sign(\gamma) = -sign(\rho)$, and induce division managers' beliefs $\hat{q}_d^d = \hat{q}_{d'}^d = e^{-\frac{\eta}{2}} q < q$. When $\rho = 0$, HQ are indifferent between $\gamma = \pm\beta$. Furthermore, $\beta + \gamma < 1$ for $\eta > 2\ln\frac{3}{2}$.

When uncertainty is low, $\eta \leq \bar{\eta}$, uncertainty aversion does not significantly affect beliefs and, thus, incentives to exert effort. At these low levels of uncertainty, the disagreement between division managers and HQ is relatively small, with $\hat{q}_d^d = e^{-\eta} q < q$, corresponding to case (i) in Lemma 2. In this case, the benefits of hedging uncertainty are too small relative to its cost (due to the increased risk exposure) and optimal incentive contracts mirror those in Lemma 1. The main difference is that the presence of uncertainty, by increasing the cost of incentive provision, reduces both pay-for-performance sensitivity and effort. The threshold level $\bar{\eta}(r,\rho)$ is increasing in both division manager risk aversion, r, and correlation coefficient ρ, making this case more relevant when risk hedging is more valuable. Cross-division exposure is $\gamma = -\rho\beta$, as in the no-uncertainty case.

When uncertainty is sufficiently large, $\eta > \bar{\eta}$, HQ finds it optimal to hedge division managers'

[27] Closed-form solutions for optimal incentive contracts can be obtained when divisions are symmetric, and condition (S) holds. They are displayed in the online supplemental materials: see Equations (B24), (B38), and (B52).

uncertainty with greater cross-division, setting $|\gamma| = \beta > |\rho|\,\beta$. The presence of large uncertainty, if left unchallenged, would significantly depress effort. By granting greater cross-division pay, HQ limits division managers' pessimism, leading to $\hat{q}^d_{d'} = e^{-\frac{\eta}{2}}q$ (corresponding to case (ii) in Lemma 2). More favorable beliefs promote effort, but at the cost of greater risk exposure. To hedge division manager risk, the sign of the cross-division exposure γ is the opposite of the sign of the correlation coefficient, with $sign\,(\gamma) = -sign\,(\rho)$.[28] When the cash flows of the two divisions are uncorrelated, cross-division exposure produces no risk-hedging benefit, only uncertainty hedging, and HQ is again indifferent between setting $\gamma = \pm\beta$.

4 Uncertainty-averse principal

Different from the case of uncertainty-neutral principal, beliefs held by uncertainty-averse HQ are not fixed but, rather, are determined endogenously as well. HQ beliefs $\hat{q}^{HQ} = \{\hat{q}^{HQ}_A, \hat{q}^{HQ}_B\}$ solve

$$\min_{\hat{q}^{HQ}\in K^{HQ}} \pi(\hat{q}^{HQ}) = \sum_{d\in\{A,B\}} E\left[Y_d(a_d) - w_d\,(Y)\,|\hat{q}^{HQ}\right] \tag{22}$$

where

$$K^{HQ} \equiv \left\{ \hat{q}^{HQ} | \ln\left(\frac{1}{1-\left|\frac{\hat{q}^{HQ}_A - q_A}{q_A}\right|}\right) + \ln\left(\frac{1}{1-\left|\frac{\hat{q}^{HQ}_B - q_B}{q_B}\right|}\right) \leq \eta^{HQ} \right\}. \tag{23}$$

The following lemma characterizes HQ beliefs for the case in which HQ has positive residual exposure in either division, $\beta_d + \gamma_{d'} < 1$. We will later show that this holds for suffiently large uncertainty (specifically, if $\eta > \eta^{HQ} + 2\ln\frac{3}{2}$).

Lemma 4 *Let $\beta_d + \gamma_{d'} < 1$, $d \in \{A, B\}$ with $d' \neq d$, and*

$$H^{HQ}_d \equiv \frac{(1-\beta_{d'} - \gamma_d)a_{d'}q_{d'}}{(1-\beta_d - \gamma_{d'})a_d q_d}, \tag{24}$$

Headquarters assessment of both divisions, $(\hat{q}^{HQ}_A, \hat{q}^{HQ}_B)$, is equal to:

(i) $\hat{q}^{HQ}_d = \left[e^{-\eta^{HQ}} H^{HQ}_d\right]^{\frac{1}{2}} q_d$ *for* $H^{HQ}_d \in \left[e^{-\eta^{HQ}}, e^{\eta^{HQ}}\right]$

(ii) $\hat{q}^{HQ}_d = q_d$ *and* $\hat{q}^{HQ}_{d'} = e^{-\eta^{HQ}} q_{d'}$ *for* $H^{HQ}_d > e^{\eta^{HQ}}$.

Similar to Lemma 2, HQ beliefs depend on its relative exposure to both divisions, as measured by the corresponding hedging ratio H^{HQ}_d (note that $H^{HQ}_{d'} = 1/H^{HQ}_d$). When HQ has a balanced

[28] Note that, when HQ is uncertainty neutral, equity-based compensation is not optimal with positively correlated cash flows. In the next section, we show that equity based compensation is optimal when HQ is uncertainty averse and there is sufficient uncertainty.

exposure to the two divisions, as in case (i) with $H_d^{HQ} \in \left[e^{-\eta^{HQ}}, e^{\eta^{HQ}}\right]$, HQ has conservative beliefs toward each division, $\hat{q}_d^{HQ} < q_d$, and is less confident toward a division as relative exposure to that division increases. When HQ exposure to a division is sufficiently large, as in case (ii) with $H_d^{HQ} > e^{\eta^{HQ}}$, HQ is even less confident about that division, $\hat{q}_d^{HQ} = e^{-\eta} q_d$, and correspondingly more confident on the other division, $\hat{q}_{d'}^{HQ} = q_{d'}$. Beliefs for division managers are still given as in Lemma 2, and their effort levels as in Lemma 3.

4.1 Uncertainty and relative-performance compensation

To separate the effect of uncertainty aversion and risk aversion on optimal incentive contracts, we start again with the simpler case where both HQ and division managers are uncertainty averse but risk neutral. For expositional simplicity and tractability, we focus on the case where division managers' uncertainty is the same, $\eta^A = \eta^B = \eta$, and HQ faces sufficiently large uncertainty (that is, large η^{HQ}).[29]

Theorem 3 *(Uncertainty and relative-performance compensation) Let both HQ and division managers be uncertainty averse but risk neutral. If HQ is sufficiently uncertainty averse,* $\eta^{HQ} > \max_{d \in \{A,B\}} \ln \hat{H}_d$, *where* $\hat{H}_d \equiv (\theta_{d'}/\theta_d)^{1/2} q_{d'}/q_d$, *and division managers are sufficiently more uncertainty averse than HQ,* $\eta > \eta^{HQ} + 2\ln\frac{3}{2}$, *optimal contracts have* $H_d^{HQ} = H_d = \hat{H}_d$ *with pure equity:*

$$\beta_d = \gamma_d = \frac{1}{1 + 3(1 - \hat{q}_d^d / \hat{q}_d^{HQ})} < 1 \tag{25}$$

with $\beta_d = \gamma_d < 1$. *Division managers are more pessimistic than HQ:* $\hat{q}_d^d < \hat{q}_d^{HQ}$.

When there is sufficient uncertainty, $\eta^{HQ} > \ln \hat{H}_d$ and $\eta > \eta^{HQ} + 2\ln\frac{3}{2}$, optimal incentive contracts are pure equity, $\beta_d = \gamma_d$.[30] Pay-for-performance sensitivity and effort levels mimic those in Theorem 1, with the difference that now HQ beliefs are endogenous and equal $\hat{q}_d^{HQ}$ rather than q_d, as in (20). Absent risk aversion, optimal contracts provide uncertainty sharing: HQ equates its uncertainty-hedging ratio with respect to each division to the uncertainty-hedging ratio of its division manager by setting $H_d^{HQ} = H_d$.

[29] It is possible, although messy, to extend the analysis to the case in which division managers are exposed to different levels of uncertainty, $\eta_A \neq \eta_B$. The optimal contract in Theorem 3 is still equity, $\beta_d = \gamma_d$, but division managers receive different equity shares: $\beta_A \neq \beta_B$.

[30] These conditions ensure that HQ has a positive exposure to both divisions, $1 - \beta_d - \gamma_{d'} > 0$, and that its beliefs fall in case (ii) of Lemma 4. For smaller values of η^{HQ}, HQ sets $\hat{q}_d^{HQ} = e^{-\eta^{HQ}} q_d$ at the larger division and $\hat{q}_{d'}^{HQ} = q_{d'}$ at the smaller. Optimal contracts will be as in Section 3.

From (25), pay-for-performance sensitivity, β_d, and cross-pay, γ_d, now depend on the difference in beliefs held by HQ and the division manager, $\hat{q}_d^d/\hat{q}_d^{HQ}$, where such difference depends on HQ uncertainty relative to that of the division manager. An increase of HQ uncertainty increases pay-for-performance sensitivity, cross-pay, and effort. This happens because, when $\eta^{HQ} < \eta$, an increase of HQ uncertainty, η^{HQ}, worsens its beliefs estimates, $\hat{q}_d^{HQ}$, and brings its beliefs closer to the division manager's beliefs, $\hat{q}_d^d$, reducing the uncertainty discount. The smaller discount lowers the cost of incentive provisions and induces HQ to offer contracts with larger pay-for-performance sensitivity, β_d, leading to greater effort. Greater pay-for-performance sensitivity, however, increases a division manager's risk exposure, which is offset by a corresponding increase of cross-pay, γ_d.

Importantly, HQ uncertainty aversion affects the desirability of relative-performance compensation. With uncertainty-neutral HQ, Theorem 1 shows that $|\gamma_d| = \xi_d \beta_d$, which implies that HQ is indifferent with respect to relative-performance compensation versus equity-based compensation with identical cross-division exposure (in absolute value). The presence of uncertainty aversion by HQ introduces an additional source of disagreement with division managers, breaking this indifference. This happens because relative-performance compensation for division manager d (by setting $\gamma < 0$) generates a "short" exposure to the other division, d', while HQ still holds a "long" position in that division, because $1 - \beta - \gamma > 0$. From Lemma 4, when HQ is uncertainty averse and holds a long position in d', it is more pessimistic than the reference probability, $\hat{q}_{d'}^{HQ} < q$. In contrast, from Lemma 2, division managers holding a short position, $\gamma < 0$, are more confident about the other division d' than the reference probability, $\hat{q}_{d'}^d \geq q$. The combined effect is that HQ and division managers now hold more divergent views on the value of compensation contracts, increasing the uncertainty discount and the cost of hedging risk. This increases the cost of relative-performance contracts, making equity-based contracts optimal by setting $\beta_d = \gamma_d$.

4.2 Risk, uncertainty, and cross-pay

The presence of division managers' risk aversion again affects optimal contracts. When division managers are risk averse, full equity exposure, $\beta_d = \gamma_d$, may lead to suboptimal risk sharing, especially when division cash flows are positively correlated (a case where relative-performance contracts, $\gamma_d < 0$, would offer a risk-hedging benefit). With risk-averse division managers, optimal cross-division exposure γ_d depends again on the trade-off between costs and benefits of hedging division manager risk and uncertainty (with their effects on effort provision). Optimal cross-division exposure is characterized in the following theorem.

Theorem 4 *(Uncertainty and cross-pay) Let HQ be uncertainty averse, while division managers are both uncertainty and risk averse, with* $\eta > \eta^{HQ} + 2\ln\frac{3}{2}$*. Let* (γ_d, β_d) *be the optimal contract and condition (S) holds. There is a threshold* $\hat{\eta}$ *such that cross-division exposure* γ_d *is as follows:*

(i) If $\eta \leq \hat{\eta}$*, there is a* $\hat{\eta}^{HQ}$ *such that the optimal contract has*

(a) $\gamma = -(\rho - \bar{\rho})\beta$ *if* $\eta^{HQ} < \hat{\eta}^{HQ}$*, where* $\bar{\rho} \equiv \hat{q}_d^d \left(\hat{q}_{d'}^d - \hat{q}_{d'}^{HQ}\right) \frac{\theta}{r\sigma^2}$*, and*

(b) $\gamma = \beta$ *if* $\eta^{HQ} \geq \hat{\eta}^{HQ}$*.*

(ii) If $\eta > \hat{\eta}$ *the optimal contract has:*

(a) for $\rho \leq 0$*, equity contracts are optimal:* $\gamma = \beta$*;*

(b) for $\rho > 0$*, there are thresholds* $(\hat{\eta}_1^{HQ}, \hat{\eta}_2^{HQ})$*, with* $\hat{\eta} < \hat{\eta}_1^{HQ} < \hat{\eta}_2^{HQ}$*, and a* $\hat{\xi}(\eta^{HQ})$*, such that*

(b.1) $\gamma = -\hat{\xi}(\eta^{HQ})\beta$*, if* $\eta^{HQ} < \hat{\eta}_1^{HQ}$*, and*

(b.2) $\gamma = \beta$ *if* $\eta^{HQ} \geq \hat{\eta}_2^{HQ}$*.*

$\hat{\xi}(\eta^{HQ}) \in (e^{-\eta}, 1)$ *is increasing in* r*,* σ*,* η*, and decreasing in* θ*,* q*, and* η^{HQ}*, with* $\hat{\xi}(0) = 1$*. The hedging factor* $\hat{\xi}(\eta^{HQ})$ *and thresholds* $\hat{\eta}, \hat{\eta}^{HQ}, \hat{\eta}_1^{HQ}, \hat{\eta}_2^{HQ}$ *are defined in the appendix,*

When both division managers' and HQ's uncertainty is sufficiently low as in case (i)(a), with $\eta \leq \hat{\eta}$, and $\eta^{HQ} < \hat{\eta}^{HQ}$, optimal cross-division exposure, γ, mirrors that absent uncertainty of Lemma 1. The important difference is that relative-performance compensation, $\gamma < 0$, is now optimal only with sufficiently large positive correlation, $\rho > \bar{\rho} \geq 0$ (note that $\bar{\rho} = 0$ when $\eta^{HQ} = 0$). The reason is that the presence of uncertainty aversion, HQ increases the uncertainty discount and raises the cost of hedging division manager risk with relative-performance compensation.

The implication is that relative-performance compensation is optimal only when the correlation between divisions and, thus, the risk-hedging benefits are sufficiently large; that is, when $\rho > \bar{\rho}$. The threshold $\bar{\rho}$ is a decreasing function of a division's risk and of division managers' risk aversion (which both increase the benefits of hedging risk), and is an increasing function of division size (which increases HQ exposure to a division's uncertainty, exacerbating the uncertainty discount). The threshold $\bar{\rho}$ is also an increasing function of the disagreement between division managers and HQ on cross-division productivity, $\hat{q}_{d'}^d - \hat{q}_{d'}^{HQ}$. Greater disagreement increases the uncertainty discount and increases the cost to HQ of relative-performance compensation, making it desirable only when the benefits of risk hedging are sufficiently large.

When HQ is exposed to large uncertainty, as in case (i)(b) with $\eta^{HQ} > \hat{\eta}^{HQ}$, optimal incentive contracts are again pure equity, $\beta = \gamma$, with no relative-performance compensation even with positively correlated cash flows. Large uncertainty exacerbates disagreement on relative-performance compensation and results in a more significant cost of hedging division-manager risk. In this situation, hedging uncertainty conflicts with hedging risk: the uncertainty-hedging motive overcomes the risk-hedging motive, and HQ foregoes altogether the risk-hedging benefits of relative-performance compensation. Rather, it offers pure-equity contracts that better align division managers' beliefs with those of HQ, lowering the cost of incentive provision and promoting effort. This case is an important reversal of the predictions made in Lemma 1 of the standard contracting problem with no uncertainty.

When division managers face sufficiently large uncertainty, as in case (ii) with $\eta > \hat{\eta}$, cross-division exposure γ depends on the sign of the correlation coefficient. If division cash flows are negatively correlated, as in case (ii)(a) with $\rho \leq 0$, then optimal contracts are again pure equity, with $\beta_d = \gamma_d$. In this situation, uncertainty hedging and risk hedging are synergetic, making equity-based compensation optimal.[31]

When division cash flows are positively correlated and HQ is exposed to low levels of uncertainty, as in case (ii)(b.1) with $\eta^{HQ} \leq \hat{\eta}_1^{HQ}$, optimal contracts have a relative-performance component, with $\gamma < 0$. Cross-division exposure is again proportional to pay-for-performance sensitivity by a factor $\hat{\xi}$, which represents the hedging component of division manager compensation. Importantly, the hedging factor $\hat{\xi}$ depends now on the level of division managers' risk aversion and their exposure to uncertainty, relative to the uncertainty faced by HQ. Greater managerial risk aversion and cash flow risk, σ, increase the importance of hedging the division manager's risk, leading to more cross-division exposure (larger $\hat{\xi}$). Similarly, greater uncertainty aversion by division managers increases the importance of uncertainty hedging, leading again to more cross-division exposure. In contrast, greater uncertainty by HQ (greater η^{HQ}) and greater division uncertainty (larger values of θ and q), by exacerbating the uncertainty discount, increase the cost of both risk hedging and uncertainty hedging, leading to a lower hedging factor $\hat{\xi}$. Finally, when HQ faces sufficiently large uncertainty, as in case (ii)(b.2) with $\eta^{HQ} \geq \hat{\eta}_2^{HQ}$, the disagreement discount overwhelms again the risk-hedging

[31] Interestingly, in the appendix we show that if HQ and division managers face the same uncertainty, $\eta_{HQ} = \eta$, they then share the same vision in the firm, $\hat{q}_d^d = \hat{q}_{d'}^d = \hat{q}_d^{HQ} = e^{-\frac{\eta}{2}} q$. Equity-based compensation has the desirable effect of coordinating internal beliefs in the organization, achieving consensus.

benefits and optimal incentive contracts offer equity-based compensation, $\gamma = \beta$.

Importantly, Theorem 4 shows that when uncertainty is sufficiently large, optimal incentive contracts include equity components and no relative-performance components, even in the case of positively correlated division cash flows. Positive correlation of divisional cash flows is particularly relevant in practice because it may reflect exposure to common aggregate risk factors, such as the business cycle. This result is, again, in sharp contrast to the standard optimal contracts absent uncertainty aversion.

5 Extensions and discussion

5.1 Optimal uncertainty hedging

One of the main results of our paper is to establish the benefits of uncertainty hedging through either equity-based or relative-performance compensation for incentive provision. HQ, however, can more generally hedge division manager uncertainty by making incentive contracts depend on additional hedging variables that are external to a firm, such as appropriate benchmarks.[32] This section will show the superiority of contracting with hedging variables internal to the firm over similar variables that are external to the firm.

We modify the basic model as follows. For simplicity, HQ contracts only with the manager of division A. Division B is not affected by agency problems and, thus, does not create a contracting problem; for simplicity, it has no division manager. We now assume that HQ can contract with division manager A on an additional (external) hedging variable C, to which HQ has no direct exposure. For simplicity, we consider the case where both B and C are uncorrelated[33] with A, and that B and C have identical mean, μ, and the same variance as division A, given by σ^2.

An incentive contract is now a triplet $\{\beta, \gamma, \psi\}$ specifying the exposure to the manager's division, β, the exposure to the other division, γ, and the exposure to the external hedge, ψ. The division manager and HQ are uncertainty averse. Because the triplet $\{A, B, C\}$ is uncertain, the core beliefs set is

$$K^i = \left\{ \hat{q} \middle| \ln\left(\frac{1}{1 - \left|\frac{\hat{q}_A - q_A}{q_A}\right|}\right) + \ln\left(\frac{1}{1 - \left|\frac{\hat{q}_B - q_B}{q_B}\right|}\right) + \ln\left(\frac{1}{1 - \left|\frac{\hat{q}_C - q_C}{q_C}\right|}\right) \leq \eta^i \right\} \tag{26}$$

[32] Examples of external hedging variables include equity in firms in the same industry (for example, shares of Lyft for an executive at Uber), an industry index, and a commodity index. Inclusion of equity of competitors raises strategic concerns due to common ownerships that we ignore.

[33] If A, B, and C are correlated, but B and C are not perfectly correlated, then it is optimal to hedge with all three even in the absence of uncertainty, as required by the informativeness principle.

for $i \in \{A, HQ\}$. Payoff to HQ is

$$\check{\Pi} = \min_{\hat{q}^{HQ} \in K^{HQ}} \check{\pi} \equiv (1-\beta)\, a_A \hat{q}_A^{HQ} + (1-\gamma)\, \mu \hat{q}_B^{HQ} - \psi \mu \hat{q}_C^{HQ} - s. \tag{27}$$

Division manager has payoff

$$\check{U} = \min_{\hat{q}^{A} \in K^{A}} \check{u} \equiv s + \beta a_A \hat{q}_A^{A} + \gamma \mu \hat{q}_B^{A} + \psi \mu \hat{q}_C^{A} - \frac{r\sigma^2}{2}\left(\beta^2 + \gamma^2 + \psi^2\right) - \frac{a_A^2}{2\theta_A}. \tag{28}$$

We first ask the preliminary question: if HQ hedges division manager uncertainty with only one hedging variable, either B or C, does it prefer to contract on the internal or external hedge?

Lemma 5 *(Internal vs external hedges) HQ prefers to hedge division manager uncertainty using internal rather than external hedges. If HQ considers granting contract $(\beta, 0, \psi)$, then it will weakly prefer contract $(\beta, |\psi|, 0)$.*

HQ prefers to hedge division manager uncertainty using internal rather than external hedges; that is, to use B rather than C. Hedging uncertainty by granting exposure to another division has an advantage over hedging uncertainty through exposure to an external hedge. The reason is that the latter leads HQ and division managers to hold opposite positions, whereby one party holds a long position and the other party holds a short position on the hedge. The effect is that the party holding the long position will be more pessimistic and the party holding the short position will be more optimistic. The difference of beliefs will lead to disagreement as to the value of the hedges, making it more costly (in the eyes of HQ) to meet the division manager participation constraint. In contrast, with an internal hedge, HQ holds a residual claim on the cash flows of all divisions, alleviating the disagreement and, thus, the cost of hedging uncertainty. When uncertainty faced by HQ is sufficiently large, external hedges become undesirable, as established in the following theorem.

Theorem 5 *(Optimal internal hedges) If $\eta^{HQ} > \breve{\eta}^{HQ}$, then incentive contracts are equity with no external hedges: $\beta = \gamma$, and $\psi = 0$.*

With low levels of uncertainty, including external hedges in incentive contracts (such as industry benchmarks) in combination with internal hedges (such as cross-division exposure) may still be beneficial because they improve the overall effectiveness of incentive contracts. The potential benefit to hedging uncertainty by including external hedges, however, must be balanced against two costs. The first cost is the additional risk exposure imposed on division managers, which must be compensated by a corresponding risk premium. The second cost is that of the hedge itself. High levels of uncertainty increase the disagreement between HQ and division managers, leading them to hold sharply different beliefs. Greater disagreement is costly because it exacerbates the uncertainty discount, making it more difficult for HQ to meet division managers' participation constraint. When uncertainty is sufficiently large, that is for $\eta^{HQ} > \breve{\eta}^{HQ}$, the uncertainty discount overwhelms the benefits of using external hedges, which consequentially are not included in incentive contracts. An implication of Theorem 5 is that internal hedges, represented by exposure to uncertain variables that are already on a company's balance sheet, represent "natural hedges" and have an advantage with respect to external hedges for hedging uncertainty.

5.2 The case of rectangular beliefs

An important assumption in our paper is that the core beliefs set is a strictly convex set with smooth boundaries, which guarantees that beliefs respond to changes in compensation contracts. While this property is satisfied by the relative entropy criterion, it does not hold when agents hold "rectangular" beliefs (as in Equation 3.11 of Chen and Epstein, 2002), such as for

$$K^i(q^i) \equiv \{\hat{q}^i : [q_A - \eta^i \leq \hat{q}^i_A \leq q_A + \eta^i] \times [q_B - \eta^i \leq \hat{q}^i_B \leq q_B + \eta^i]. \tag{29}$$

With rectangular beliefs, uncertainty-averse agents do not benefit from uncertainty hedging. In the context of our paper, division manager and HQ beliefs on division productivity – the solutions to (17) and (22) – are determined by a fixed "worst-case scenario" and do not depend on the relative exposure to division uncertainty generated by incentive contracts.[34]

The presence of rectangular beliefs, however, exacerbates the uncertainty discount for relative-performance contracts, strengthening the results of Section 4.1. This happens because relative-performance contracts, where division managers and HQ have opposite exposures to cross-division cash flow, lead them to hold extreme opposite beliefs on division productivity. HQ, by holding a long position in both divisions, sets beliefs at the lower extreme of the belief range, $q_i^{HQ} - \eta^{HQ}$. In contrast, division managers, by holding a short position in the cross-division cash flow, are concerned when that division has high productivity, and set beliefs at the higher extreme of the belief range, $q^d_{d'} + \eta^d$. Thus, rectangular beliefs lead to extreme disagreement between HQ and

[34] Intuitively, the solutions to the minimization problems in the right-hand side and left-hand side of Equation (2) are equivalent, and the condition holds as an equality.

division managers, exacerbating the uncertainty discount. The effect is to make it even more costly, from the point of view of HQ, to meet division managers' participation constraint, making relative-performance compensation less desirable. When HQ uncertainty, η^{HQ}, is sufficiently large, optimal contracts have no cross-pay:

Theorem 6 *Let $\rho > 0$; there is a threshold η_1^{HQ} such that if $\eta^{HQ} > \eta_1^{HQ}$ then the optimal contract has no cross-pay with $\gamma_d = 0$. The optimal pay-performance sensitivity is*

$$\beta_d = \frac{1}{1 + \left(1 - \frac{\hat{q}_d^d}{\hat{q}_d^{HQ}}\right) + \frac{r\sigma^2}{\theta \hat{q}_d^{HQ} \hat{q}_d^d}}. \tag{30}$$

5.3 Mergers and synergies

Equity-based compensation may also be desirable in the presence of synergies, which we have ruled out so far. Synergies create an externality among division managers, leading to a moral hazard in team problem (as in Holmström, 1982). We introduce the externality due to synergies by assuming that divisional output depends now on the effort exerted by both division managers, $\mu_d = (a_d + \zeta a_{d'})\, q_d$, where the parameter ζ represents the intensity of the synergy (the externality).

Theorem 7 *Let condition S hold. For any level of division manager uncertainty η, there is a threshold of synergy intensity $\bar{\zeta}$ such that for all $\zeta > \bar{\zeta}$, the optimal contract is equity: $\gamma = \beta$. When $\eta = 0$, pure equity is optimal only with $\zeta = 1$.*

The presence of synergies strengthens the incentives to adopt equity-based compensation. In particular, for any level of uncertainty, η, pure equity-based compensation is optimal if synergy intensity is sufficiently large, $\zeta > \bar{\zeta}$. Interestingly, absent uncertainty, $\eta = 0$, equity-based compensation is optimal when division manager effort are perfect substitutes, $\zeta = 1$.

We conclude the section by noting that, from Lemma 5, adding a division to a firm is desirable for its beneficial effect of creating additional uncertainty-hedging opportunities. In this way, uncertainty hedging provides a new source of value in mergers.[35]

6 Empirical implications

Our paper offers several empirical implications that can help explain some otherwise puzzling features of the compensation policies adopted by corporations.

[35] This feature mirrors the equivalent property based on risk diversification alone: adding a division to a firm is beneficial because it allows a firm to reduce overall cash flow volatility, reducing (for example) expected bankruptcy costs (see Lewellen, 1971). In this case, if division cash flows are uncorrelated, then firms could fully exploit the benefits of diversification.

1. *Firms characterized by high uncertainty, such as young firms, prefer compensation contracts with an equity rather than relative-performance component.* A puzzling feature of the compensation structure of many young firms is the widespread use of equity-based compensation throughout the organization, even for lower-level managers and rank-and-file employees. Our paper provides an explanation for the optimality of equity-based compensation and the infrequent use of relative-performance assessments. We argue that equity-based compensation provides two important benefits. First, it better aligns the beliefs of members of the organization with the one held by top management. Absent the equity component in pay, individuals would hold more conservative beliefs than top management on the expected performance of their unit. Inclusion of equity-based compensation improves employee expectations about firm profitability, leading to greater effort and, thus, firm value. The second benefit of equity-based compensation is to align employee expectations with the ones held by top management, improving overall disposition in the organization.

In contrast, relative-performance compensation exacerbates disagreement within the organization, with two adverse effects: it increases the cost of incentive provision and creates discord within the organization. We show that when uncertainty is sufficiently high (such as for young firms engaged in new technologies), optimal incentive contracts offer equity-based compensation, irrespective of the correlation across divisions.

As firms mature, the level of uncertainty surrounding their business activities decreases, reducing (or even eliminating) the need for equity-based compensation. For these firms, effort levels in the organization are better elicited by the use of pay-for-performance incentive contracts, making equity-based compensation redundant. This means that firms should first start, when they are young, with incentive contracts skewed heavily toward equity-based compensation, and then move toward pay-for-performance based contracts as they mature.

2. Benchmarking and pay-for-luck. It is often suggested that lack of relative-performance components in executive pay (i.e., benchmarking) rewards top managers for performance influenced by market forces outside their control rather than their own effort ("pay-for-luck").[36] Our paper suggests that hedging managerial risk exposure through benchmarking can be costly. Benchmarking creates a divergence between top executives, who hold "short" positions in the benchmarks, and shareholders, who would hold a "long" position. The effect is that managers value their compen-

[36] Gopalan et al. (2010) argue this is a response to strategic uncertainty surrounding firms.

sation contracts at a discount, making it more costly to meet their participation constraint.

3. Optimal compensation in business groups. Our paper also offers implications for the compensation structure in business groups. Traditional theory suggests that compensation of managers in subsidiaries of a business (or family) group should depend only on the performance of their business units. In contrast, compensation for such managers is often tied to the performance of the entire business group. For example, Ma et al. (2019) study the compensation structure for the mutual funds industry and find that in about half of their sample, managers' bonuses are directly linked to the overall profitability of the advisor. A similar practice is common in the investment bank industry, where individual bonuses depend also on the overall performance of the intermediary. Such features, which would be difficult to explain on the basis of risk-aversion only, are consistent with the findings of our paper.

4. *Managerial (over)optimism.* Our model predicts that managers in the upper echelon of corporate ladders tend to be relatively more optimistic about their firm's future performance. This implies that, rank-and-file managers perceive members of the top management team of their firm (such as CEOs and CFOs) as overconfident and unrealistically optimistic. The presence of managerial overconfidence in corporations has been extensively documented (see, Heaton, 2002, and Malmendier and Tate, 2005, among others).[37] We suggest that top managers' optimism can be the consequence of uncertainty hedging, and not necessarily the sign of a behavioral bias.

7 Conclusions and future research

We examine the impact of uncertainty aversion on the design of optimal incentive contracts in an organization. We show that, by proper design of compensation contracts, firms can affect employee expectations, with a positive effect on incentives. This feature suggests that compensation contracts can affect the structure of beliefs within the organization. Equity-based compensation can realign internal beliefs, promoting a shared view and internal consensus. In contrast, relative-performance compensation may lead division managers to be more confident about other divisions in the firm, relative to theirs, creating envy and discord. Such discord may interfere with the overall management and performance of the organization, for example by affecting the internal allocation of resources. We leave the exploration of these issues, and implications for organization design, to

[37] Goel and Thakor (2008) suggest that managerial optimism can be the outcome of the managerial selection process, whereby lucky and overconfident managers are more likely to rise to the top positions of companies.

future research.

The analysis in our paper can be extended in several ways. It would be interesting to examine multitasking situations, as in Holmström and Milgrom (1991). Our paper suggests an important aspect of uncertainty hedging and its impact on task assignment and optimal compensation. An additional avenue of research is to determine the impact of uncertainty on organization design; it is plausible to expect that organizations in highly uncertain environments have a relatively flat structure, to promote uncertainty hedging. Our paper is a partial equilibrium model; an interesting question is to examine the impact of labor market forces in a process where heterogenous agents are matched with heterogenous firms. We leave these important questions for future research.

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Appendix: Proofs

Proof of Lemma 1. Each division manager d selects a_d to maximize (5). Because $K^i(q) = q$ and $c(a) = \frac{a_d^2}{2\theta_d}$, $\frac{du_d}{da_d} = \beta_d q_d - \frac{a_d}{\theta_d}$. Because (5) is strictly concave, the first-order condition is sufficient for a maximum: $a_d = \beta_d \theta_d q_d$. The participation constraint, (8), binds at the optimum: substituting (9) into (6), $\hat{\pi} = \sum_{d \in \{A,B\}} \left[q_d a_d - \frac{r\sigma^2}{2}\left(\beta_d^2 + 2\rho\beta_d\gamma_d + \gamma_d^2\right) - c_d(a_d) \right]$. Because γ_d does not affect a_d, $\frac{\partial \hat{\pi}}{\partial \gamma_d} = -r\sigma^2(\rho\beta_d + \gamma_d)$, so it is optimal to set $\gamma_d = -\rho\beta_d$. Substituting $a_d = \beta_d\theta_d q_d$ in $\hat{\pi}$ and differentiating, we obtain (13). Second order conditions are satisfied by concavity of (6). Note that $\beta_d + \gamma_d = \frac{(1-\rho)}{1+r\sigma^2\frac{1-\rho^2}{\theta q^2}}$, so $\beta_d + \gamma_d < 1$ iff $r\sigma^2 > -\frac{\rho\theta q^2}{1-\rho^2}$. ■

Proof of Lemma 2. In this proof, we will focus on the case (ii) when $H_d \in \left(e^{-\eta^d}, e^{\eta^d}\right)$ and $\gamma_d > 0$ (other cases presented in the supplemental materials). Consider $\tilde{q}_d^d = q_d + \delta$, for $\delta > 0$. Switching to $\tilde{q}_d^{d-} = q_d - \delta$ lowers $\hat{u}_d$ by $2\beta_d a_d \delta$ while leaving the constraint unchanged. Similarly, switching from $\tilde{q}_{d'}^d = q_{d'} + \delta$, for $\delta > 0$ to $\tilde{q}_{d'}^{d-} = q_{d'} - \delta$ lowers $\hat{u}_d$ by $2\gamma_d a_{d'}\delta$, leaving the constraint unchanged. Therefore, it must also be that $\hat{q}_d^d \leq q_d$ and $\hat{q}_{d'}^d \leq q_{d'}$: the division manager will be pessimistic toward both sources of risk. On case (ii), these will hold strictly, $\hat{q}_d^d < q_d$ and $\hat{q}_{d'}^d < q_{d'}$, so the Lagrangian for the relaxed problem is

$$\mathcal{L} \equiv -\hat{u}_d - \lambda\left[g_c - \eta^d\right] \tag{A1}$$

where $g_c \equiv \ln\frac{q_d}{\hat{q}_d^d} + \ln\frac{q_{d'}}{\hat{q}_{d'}^d}$. Note $\frac{\partial \mathcal{L}}{\partial \hat{q}_d^d} = -\beta_d a_d + \frac{\lambda}{\hat{q}_d^d}$ and $\frac{\partial \mathcal{L}}{\partial \hat{q}_{d'}^d} = -\gamma_d a_{d'} + \frac{\lambda}{\hat{q}_{d'}^d}$. Because $\frac{\partial \mathcal{L}}{\partial \hat{q}_d^d} = 0$ but $\beta_d a_d > 0$, it must be that $\lambda > 0$. By complementary slackness, $g_c = \eta^d$. Therefore, $\frac{\partial \mathcal{L}}{\partial \hat{q}_d^d} = 0$ iff $\lambda = \beta_d a_d \hat{q}_d^d$, while $\frac{\partial \mathcal{L}}{\partial \hat{q}_{d'}^d} = 0$ iff $\lambda = \gamma_d a_{d'} \hat{q}_{d'}^d$. Because $g_c = \eta^d$ implies that $\hat{q}_d^d \hat{q}_{d'}^d = e^{-\eta^d} q_d q_{d'}$, or equivalently, $\hat{q}_d^d = [e^{-\eta_d} H_d]^{\frac{1}{2}} q_d$, where $H_d = \frac{|\gamma_d| a_{d'} q_{d'}}{\beta_d a_d q_d}$. ■

Outline of Proof of Lemma 3. The lemma is shown in two steps. First, we obtain division managers' best response functions, $a_d = \theta_d \beta_d \hat{q}_d^d$, as function of their beliefs, as in Lemma 2. Second, because $\hat{q}_d^d$ is positive, continuous, and increasing in $a_{d'}$, we characterize the Nash equilibrium in terms of $\log(a_d)$ and we apply the contraction mapping theorem, proving uniqueness. Comparative statics follow by substituting in $\hat{q}_d^d$ from Lemma 2. ■

Proof of Theorem 1. Because (16) binds and $r = 0$, we can express HQ's payoff as

$$\hat{\pi} = \sum_{\substack{d,d' \in \{A,B\}, \\ d' \neq d}} \left(q_d a_d - \beta_d a_d \left(q_d - \hat{q}_d^d\right) - \gamma_d a_{d'} \left(q_{d'} - \hat{q}_{d'}^d\right) - \frac{a_d^2}{2\theta_d} \right), \tag{A2}$$

where a_d are the Nash-equilibrium effort levels of Lemma 3. We will show the optimal contract when $\gamma_d \geq 0$. In the supplemental materials, we show that the objective is symmetric in γ_d around zero. Thus, if $\left(\tilde{\beta}_d, \tilde{\gamma}_d\right)$ is an optimal contract, so is $\left(\tilde{\beta}_d, -\tilde{\gamma}_d\right)$.

If $\gamma_d > e^{\eta^d} \frac{\beta_d a_d q_d}{a_{d'} q_{d'}}$, division manager beliefs are in case (iii) of Lemma 2, with $\hat{q}^d_d = q_d$ and $\hat{q}^d_{d'} = e^{-\eta^d} q_{d'}$, giving $a_d = \beta_d \theta_d q_d$. Thus, $\frac{\partial \hat{\pi}}{\partial \gamma_d} = -a_{d'} q_{d'} \left(1 - e^{-\eta^d}\right) < 0$, and setting $\gamma_d > e^{\eta^d} \frac{\beta_d a_d q_d}{a_{d'} q_{d'}}$ is not optimal. Similarly, if $\gamma_d < e^{-\eta^d} \frac{\beta_d a_d q_d}{a_{d'} q_{d'}}$, division manager beliefs are in case (i) of Lemma 2, with $\hat{q}^d_d = e^{-\eta^d} q_d$ and $\hat{q}^d_{d'} = q_{d'}$, giving $a_d = \beta_d \theta_d e^{-\eta^d} q_d$. In addition, $\hat{q}^d_{d'} = q_{d'}$ and $\hat{q}^d_d = e^{-\eta^d} q_d$ together imply that $\frac{\partial \hat{\pi}}{\partial \gamma_d} = 0$ and it is weakly optimal to set $\gamma_d \geq e^{-\eta^d} \frac{\beta_d a_d q_d}{a_{d'} q_{d'}}$. This implies that HQ set $e^{-\eta^d} \frac{\beta_d a_d q_d}{a_{d'} q_{d'}} \leq \gamma_d \leq e^{\eta^d} \frac{\beta_d a_d q_d}{a_{d'} q_{d'}}$ and induce beliefs that are in case (ii) of 2, with $H_d \in \left[e^{-\eta^d}, e^{\eta^d}\right]$.

Define $\hat{u}_d(\check{a}_d, \hat{q}^d) \equiv u_d(\check{a}_d, \hat{q}^d) - s_d$. Because the participation constraint binds, $u_d(\check{a}_d, \hat{q}^d) = 0$, $-s_d = \hat{u}_d(\check{a}_d, \hat{q}^d) = \min_{\hat{q}^d \in K^{\hat{q}}_d} \hat{u}_d$. Thus, HQ's objective function becomes

$$\hat{\pi} = (1 - \beta_A - \gamma_B)\, \check{a}_A q_A + (1 - \beta_B - \gamma_A)\, \check{a}_B q_B + \hat{u}_A(\check{a}_A, \hat{q}^A) + \hat{u}_B(\check{a}_B, \hat{q}^B). \tag{A3}$$

where $\hat{u}_d = \beta_d \check{a}_d \hat{q}^d_d + \gamma_d \check{a}_{d'} \hat{q}^d_{d'} - \frac{\check{a}^2_d}{2\theta_d} = 0$ and $\check{a}_d = \left[e^{-\eta^d} \theta^2_d \beta_d \left|\gamma_d\right|\right]^{\frac{3}{8}} \left[e^{-\eta_{d'}} \theta^2_{d'} \beta_{d'} \left|\gamma_{d'}\right|\right]^{\frac{1}{8}} \left[q_d q_{d'}\right]^{\frac{1}{2}}$ is the Nash equilibrium given by (B16) in the proof of Lemma 3. This implies that

$$\begin{aligned} \frac{d\hat{\pi}}{d\beta_d} &= -q_d \check{a}_d + (1 - \beta_d - \gamma_{d'})\, q_d \frac{\partial \check{a}_d}{\partial \beta_d} + (1 - \beta_{d'} - \gamma_d)\, q_{d'} \frac{\partial \check{a}_{d'}}{\partial \beta_d} \\ &\quad + \frac{d\hat{u}_d(\check{a}_d, \hat{q}^d(\check{a}_d, w_d))}{d\beta_d} + \frac{d\hat{u}_{d'}(\check{a}_{d'}, \hat{q}^{d'}(\check{a}_{d'}, w_{d'}))}{d\beta_d}. \end{aligned} \tag{A4}$$

Because $\frac{\partial \hat{u}_d}{\partial \beta_d} = \check{a}_d \hat{q}^d_d$, $\frac{\partial \hat{u}_d}{\partial \check{a}_{d'}} = \gamma_d \hat{q}^d_{d'}$, and $\frac{\partial \check{a}_{d'}}{\partial \beta_d} = \frac{\check{a}_{d'}}{8\beta_d}$, by applying the envelope theorem on $\hat{u}_d(\check{a}_d, \hat{q}^d)$, $\frac{d\hat{u}_d(\check{a}_d, \hat{q}^d(\check{a}_d, w_d))}{d\beta_d} = \check{a}_d \hat{q}^d_d + \gamma_d \hat{q}^d_{d'} \frac{\check{a}_{d'}}{8\beta_d}$. Because $\frac{\partial \hat{u}_{d'}}{\partial \beta_d} = 0$, $\frac{\partial \hat{u}_{d'}}{\partial \check{a}_d} = \gamma_{d'} \hat{q}^{d'}_d$, and $\frac{\partial \check{a}_d}{\partial \beta_d} = \frac{3\check{a}_d}{8\beta_d}$, by applying the envelope theorem on $\hat{u}_{d'}(\check{a}_{d'}, \hat{q}^{d'})$, we obtain that $\frac{d\hat{u}_{d'}(\check{a}_{d'}, \hat{q}^{d'}(\check{a}_{d'}, w_{d'}))}{d\beta_d} = \gamma_{d'} \hat{q}^{d'}_d \frac{3\check{a}_d}{8\beta_d}$. Thus,

$$\begin{aligned} \frac{d\hat{\pi}}{d\beta_d} &= -\check{a}_d \left(q_d - \hat{q}^d_d\right) + (1 - \beta_d - \gamma_{d'})\, q_d \frac{3\check{a}_d}{8\beta_d} + (1 - \beta_{d'} - \gamma_d)\, q_{d'} \frac{\check{a}_{d'}}{8\beta_d} \\ &\quad + \gamma_d \hat{q}^d_{d'} \frac{\check{a}_{d'}}{8\beta_d} + \gamma_{d'} \hat{q}^{d'}_d \frac{3\check{a}_d}{8\beta_d}. \end{aligned} \tag{A5}$$

Similarly, for γ_d, by applying the envelope theorem on $\hat{u}_d(\check{a}_d, \hat{q}^d)$, we obtain that $\frac{d\hat{u}_d(\check{a}_d, \hat{q}^d(a_d, w_d))}{d\gamma_d} = \check{a}_{d'} \hat{q}^d_{d'} + \gamma_d \hat{q}^d_{d'} \frac{\check{a}_{d'}}{8\gamma_d}$. Because $\frac{\partial \hat{u}_{d'}}{\partial \gamma_d} = 0$, $\frac{\partial \hat{u}_{d'}}{\partial \check{a}_d} = \gamma_{d'} \hat{q}^{d'}_d$, and $\frac{\partial \check{a}_d}{\partial \gamma_d} = \frac{3\check{a}_d}{8\gamma_d}$, by applying the envelope theorem on $\hat{u}_{d'}(\check{a}_{d'}, \hat{q}^{d'})$, we obtain that $\frac{d\hat{u}_{d'}(\check{a}_{d'}, \hat{q}^{d'}(\check{a}_{d'}, w_{d'}))}{d\gamma_d} = \gamma_{d'} \hat{q}^{d'}_d \frac{3\check{a}_d}{8\gamma_d}$. Noting that $\frac{\partial \hat{u}_d}{\partial \gamma_d} = \check{a}_{d'} \hat{q}^d_{d'}$,

$\frac{\partial \hat{u}_d}{\partial \check{a}_{d'}} = \gamma_d \hat{q}^d_{d'}$, and $\frac{\partial \check{a}_{d'}}{\partial \gamma_d} = \frac{\check{a}_{d'}}{8\gamma_d}$,

$$\frac{d\hat{\pi}}{d\gamma_d} = -\check{a}_{d'}\left(q_{d'} - \hat{q}^d_{d'}\right) + \left(1-\beta_d-\gamma_{d'}\right) q_d \frac{3\check{a}_d}{8\gamma_d} + \left(1-\beta_{d'}-\gamma_d\right) q_{d'}\frac{\check{a}_{d'}}{8\gamma_d} + \gamma_d \hat{q}^d_{d'}\frac{\check{a}_{d'}}{8\gamma_d} + \gamma_{d'}\hat{q}^{d'}_d \frac{3\check{a}_d}{8\gamma_d}. \tag{A6}$$

Thus, from (A5) and (A6) we obtain the first-order conditions:

$$\frac{d\hat{\pi}}{d\beta_d} = -\check{a}_d\left(q_d - \hat{q}^d_d\right) + \frac{\Delta_d}{\beta_d} = 0; \quad \frac{d\hat{\pi}}{d\gamma_d} = -\check{a}_{d'}\left(q_{d'} - \hat{q}^d_{d'}\right) + \frac{\Delta_d}{\gamma_d} = 0, \tag{A7}$$

where $\Delta_d \equiv \left(1-\beta_d-\gamma_{d'}\right) q_d \frac{3\check{a}_d}{8} + \left(1-\beta_{d'}-\gamma_d\right) q_{d'}\frac{\check{a}_{d'}}{8} + \gamma_d \hat{q}^d_{d'}\frac{\check{a}_{d'}}{8} + \gamma_{d'}\hat{q}^{d'}_d\frac{3\check{a}_d}{8}$, giving

$$\beta_d \check{a}_d\left(q_d - \hat{q}^d_d\right) = \gamma_{d'}\check{a}_{d'}\left(q_{d'} - \hat{q}^d_{d'}\right). \tag{A8}$$

Because, from Lemma 2, $\beta_d \check{a}_d \hat{q}^d_d = \gamma_d \check{a}_{d'} \hat{q}^d_{d'}$, we have that (A8) implies that $\beta_d \check{a}_d q_d = \gamma_d \check{a}_{d'} q_{d'}$ and thus that $H_d = 1$, leading to $\hat{q}^d_d = \hat{q}^d_{d'} = e^{-\frac{\eta^d}{2}} q_d$ and $\check{a}_d = e^{-\frac{\eta^d}{2}}\beta_d \theta_d q_d$. Substituting the values of γ_d and $\check{a}_d$ into HQ objective, we obtain

$$\hat{\pi} = \sum_{\substack{d,d' \in \{A,B\},\\ d' \neq d}} \left[\beta_d \theta_d q_d \hat{q}^d_d - 2\beta_d^2 \theta_d \hat{q}^d_d\left(q_d - \hat{q}^d_d\right) - \frac{\beta_d^2 \theta_d \left(\hat{q}^d_d\right)^2}{2}\right], \tag{A9}$$

Differentiating, we obtain $\frac{d\hat{\pi}}{d\beta_d} = \theta_d q_d \hat{q}^d_d - 4\beta_d \theta_d \hat{q}^d_d\left(q_d - \hat{q}^d_d\right) - \beta_d \theta_d \left(\hat{q}^d_d\right)^2 = 0$, giving $\beta_d = \frac{1}{1+3\left(1-\hat{q}^d_d/q_d\right)}$. Setting $H_d = 1$ gives $\gamma_d = \frac{\check{a}_d q_d}{\check{a}_{d'} q_{d'}}\beta_d$. If divisions are symmetric, condition (S) holds, $\xi_d = 1$. Further, $\beta_d + \gamma_d = \frac{2}{1+3\left(1-\hat{q}^d_d/q_d\right)} < 1$ iff $\eta > 2\ln\frac{3}{2}$. Comparative statics follow by direct differentiation. ■

Proof of Theorem 2. Suppose that $\gamma_d > 0$ and $H_d \in \left(e^{-\eta^d}, e^{\eta^d}\right)$. This is similar to the proof of Theorem 1, except that $\frac{\partial \hat{u}_d}{\partial \beta_d} = a_d \hat{q}^d_d - r\sigma^2\left(\beta_d + \rho\gamma_d\right)$ and $\frac{\partial \hat{u}_d}{\partial \gamma_d} = \check{a}_{d'}\hat{q}^d_{d'} - r\sigma^2\left(\rho\beta_d + \gamma_d\right)$. Thus, by identical logic, $\frac{d\hat{\pi}}{d\beta_d} = -\check{a}_d\left(q_d - \hat{q}^d_d\right) - r\sigma^2\left(\beta_d + \rho\gamma_d\right) + \frac{\Delta_d}{\beta_d} = 0$ and $\frac{d\hat{\pi}}{d\gamma_d} = -\check{a}_{d'}\left(q_{d'} - \hat{q}^d_{d'}\right) - r\sigma^2\left(\rho\beta_d + \gamma_d\right) + \frac{\Delta_d}{\gamma_d} = 0$, where $\Delta_d \equiv \left(1-\beta_d-\gamma_{d'}\right) q_d \frac{3\check{a}_d}{8} + \left(1-\beta_{d'}-\gamma_d\right) q_{d'}\frac{\check{a}_{d'}}{8} + \gamma_d \hat{q}^d_{d'}\frac{\check{a}_{d'}}{8} + \gamma_{d'}\hat{q}^{d'}_d\frac{3\check{a}_d}{8}$. This implies $\beta_d \check{a}_d\left(q_d - \hat{q}^d_d\right) + r\sigma^2\beta_d^2 = \gamma_{d'}\check{a}_{d'}\left(q_{d'} - \hat{q}^d_{d'}\right) + r\sigma^2\gamma_d^2$. Lemma 2 implies $\beta_d \check{a}_d \hat{q}^d_d = \gamma_d \check{a}_{d'}\hat{q}^d_{d'}$, so this proves (21). The case when $\gamma_d < 0$ and $H_d \in \left(e^{-\eta^d}, e^{\eta^d}\right)$ is symmetric. Lastly, define $f\left(\beta_d, |\gamma_d|\right) = \beta_d a_d q_d + r\sigma^2\beta_d^2 - |\gamma_d|\, a_{d'} q_{d'} - r\sigma^2\gamma_d^2$, so that (21) holds iff $f = 0$. Suppose WLOG that $a_d q_d > a_{d'} q_{d'}$. Then, $f\left(\beta_d, \beta_d\right) = \beta_d\left(a_d q_d - a_{d'} q_{d'}\right) > 0$ and $f\left(\beta_d, \frac{a_d q_d}{a_{d'} q_{d'}}\beta_d\right) = r\sigma^2\beta_d^2\left(1 - \frac{a_d^2 q_d^2}{a_{d'}^2 q_{d'}^2}\right) < 0$. Thus, $f\left(\beta_d, |\gamma_d|\right) = 0$ implies $|\gamma_d| \in \left(\beta_d, \frac{a_d q_d}{a_{d'} q_{d'}}\beta_d\right)$ because $\frac{a_d q_d}{a_{d'} q_{d'}} > 1$. Similarly, for $a_d q_d < a_{d'} q_{d'}$ and $|\gamma_d| \in \left(\frac{a_d q_d}{a_{d'} q_{d'}}\beta_d, \beta_d\right)$. ■

Outline of Proof of Corollary 1. The proof of Corollary 1 is in the supplemental materials. Intuitively, when there is low uncertainty, $\eta \leq \bar{\eta}$, it is optimal for HQ to contract as in Lemma 1, while if there is high uncertainty, $\eta > \bar{\eta}$, it is optimal to contract to as in Theorem 1. While in Theorem 1, it did not matter if exposure to the other division was positive or negative, risk aversion and correlation breaks the tie: if $\rho < 0$, cross pay is preferred, $\gamma_d > 0$, while if $\rho < 0$, relative performance is preferred, $\gamma_d < 0$. ■

Proof of Lemma 4. Proof is same as proof of Lemma 2. ■

Outline of Proof of Theorem 3. The proof, similar to the proof of Theorem 1, is in the supplemental materials. Distinct from the proof of Theorem 1, changing from $\gamma_d < 0$ to $|\gamma_d|$ strictly improves the objective because granting $\gamma_d < 0$ results in greater disagreement, forcing HQ to pay a larger salary s_d. Further, applying the same logic as the proof of Theorem 1, we can express the first-order conditions similar to (A7):

$$\frac{d\hat{\pi}}{d\beta_d} = -\check{a}_d\left(\hat{q}_d^{HQ} - \hat{q}_d^d\right) + \frac{\Delta_d}{\beta_d} = 0, \quad \frac{d\hat{\pi}}{d\gamma_d} = -\check{a}_{d'}\left(\hat{q}_{d'}^{HQ} - \hat{q}_{d'}^d\right) + \frac{\Delta_d}{\gamma_d} = 0, \tag{A10}$$

where $\Delta_d \equiv \phi_d\hat{q}_d^{HQ}\frac{3\check{a}_d}{8} + \phi_{d'}\hat{q}_{d'}^{HQ}\frac{\check{a}_{d'}}{8} + \gamma_d\hat{q}_{d'}^{d}\frac{\check{a}_{d'}}{8} + \gamma_{d'}\hat{q}_d^{d'}\frac{3\check{a}_d}{8}$, so $\beta_d\check{a}_d\left(\hat{q}_d^{HQ} - \hat{q}_d^d\right) = \gamma_{d'}\check{a}_{d'}\left(\hat{q}_{d'}^{HQ} - \hat{q}_{d'}^d\right)$. Lemma 2 implies that $\beta_d\check{a}_d\hat{q}_d^d = \gamma_{d'}\check{a}_{d'}\hat{q}_{d'}^d$, so $\beta_d\check{a}_d\hat{q}_d^{HQ} = \gamma_{d'}\check{a}_{d'}\hat{q}_{d'}^{HQ}$. However, Lemma 4 implies $\phi_d a_d\hat{q}_d^{HQ} = \phi_{d'}a_{d'}\hat{q}_{d'}^{HQ}$, which implies the optimal contract equity: $\beta_d = \gamma_d$. ■

Outline of Proof of Theorem 4. The proof of Theorem 4 is in the supplemental materials. Intuitively, when there is low uncertainty, $\eta \leq \hat{\eta}$, it will be attractive to contract similarly to Lemma 1, except that HQ is pessimistic toward the other division: because $\eta^{HQ} > 0$, $\hat{q}_{d'}^{HQ} < \hat{q}_{d'}^d$ for small γ, so HQ distorts the share of the other division due to that small disagreement: $\gamma > -\rho\beta$. When division manager uncertainty gets large enough, it because attractive to expose the division manager to the other divisions's risk. If $\rho > 0$, it may be optimal to use relative performance, $\gamma < 0$, but the disagreement cost will limit it, so that $\gamma = -\hat{\xi}\left(\eta^{HQ}\right)\beta$, where $\hat{\xi}\left(\eta^{HQ}\right) \in (e^{-\eta}, 1)$. If $\rho \leq 0$ or if η^{HQ} is big enough, equity is optimal, similar to Theorem 3: $\beta = \gamma$. ■

Outline of Proof of Lemma 5. The proof of Lemma 5 is in the supplemental materials. Intuitively, if HQ contracts on external risk, $\psi > 0$, it will be optimistic toward that source of risk, $\hat{q}_C^{HQ} \geq q_C$, while pessimistic toward their internal risk, $\hat{q}_B^{HQ} < q_B$, so it is cheaper to compensate with internal risk than with external risk. ■

Outline of Proof of Theorem 5. The proof is in the supplemental materials. Equity is optimal similarly to Corollary 1. If HQ grants exposure to the external risk, $\psi > 0$, HQ and the division manager will have opposing positions to it, leading to a disagreement cost: $\hat{q}_C^{HQ} \geq q_C \geq \hat{q}_C^A$. When HQ uncertainty, η^{HQ}, is large enough, this cost will overwhelm the benefits of uncertainty hedging, resulting in HQ optimally not contracting with the external risk, $\psi = 0$. ■

Proof of Theorem 6. HQ solves (6)-(8), but with K from (29). Because division managers must be induced to exert effort, $\beta_d > 0$, so $\hat{q}_d^d = q_d - \eta^d$. Belief towards the other division depends on contractual exposure: if $\gamma_d > 0$, $\hat{q}_{d'}^d = q_d - \eta^d$ while if $\gamma_d < 0$, $\hat{q}_{d'}^d = q_d + \eta^d$. Because HQ will have a positive exposure to both divisions, $1 - \beta_d - \gamma_d > 0$, HQ will have worst-case scenario $\hat{q}_d^{HQ} = q_d - \eta^{HQ}$. Thus, HQ has objective

$$\Pi = (1 - \beta_A - \gamma_B)\, a_A \left(q_A - \eta^{HQ}\right) + (1 - \beta_B - \gamma_A)\, a_B \left(q_B - \eta^{HQ}\right) - s_A - s_B, \tag{A11}$$

while each division manager has objective

$$U_d = s_d + \beta_d a_d \left(q_d - \eta^d\right) + \gamma_d a_{d'} \left(q_{d'} - 1_{\gamma^d > 0}\eta^d + 1_{\gamma^d < 0}\eta^d\right) - \frac{r\sigma^2}{2}\left(\beta_d^2 + 2\rho\beta_d\gamma_d + \gamma_d^2\right) - \frac{a_d^2}{2\theta_d}. \tag{A12}$$

Note that $\frac{dU_d}{da_d} = 0$ iff $a_d = \beta_d\theta_d\left(q_d - \eta^d\right)$. Substituting in the binding participation constraint, $U_d = 0$, into the objective, we can obtain first-order conditions. For $\gamma_d > 0$, $\frac{d\Pi}{d\gamma_d} = a_{d'}\left(\eta^{HQ} - \eta^d\right) - r\sigma^2\left(\rho\beta_d + \gamma_d\right) < 0$, so $\gamma_d \leq 0$. For $\gamma_d < 0$, $\frac{d\Pi}{d\gamma_d} = a_{d'}\left(\eta^{HQ} + \eta^d\right) - r\sigma^2\left(\rho\beta_d + \gamma_d\right)$. Note that $\frac{d\Pi}{d\gamma_d} > 0$ for all $\gamma_d < 0$ if $\eta^{HQ} > \eta_1^{HQ} \equiv \frac{r\sigma^2\rho\beta_d}{a_{d'}}$. Substituting in $\gamma_d = 0$ and $a_d = \beta_d\theta_d\left(q_d - \eta^d\right)$ into the objective and solving $\frac{\partial\Pi}{\partial\beta_d} = 0$ implies (30). ■

Outline of Proof of Theorem 7. The proof of Theorem 7 is in the supplemental materials. The proof is similar to the proof of Corollary 1. Synergies motivate HQ to increase exposure to the other division, lowering the risk-sharing benefits of the contract in part (i) of Corollary 1, and the diminishing the value of the relative-performance contract in part (ii) of Corollary 1. For any level of uncertainty, when synergies are large enough, equity is optimal. ■

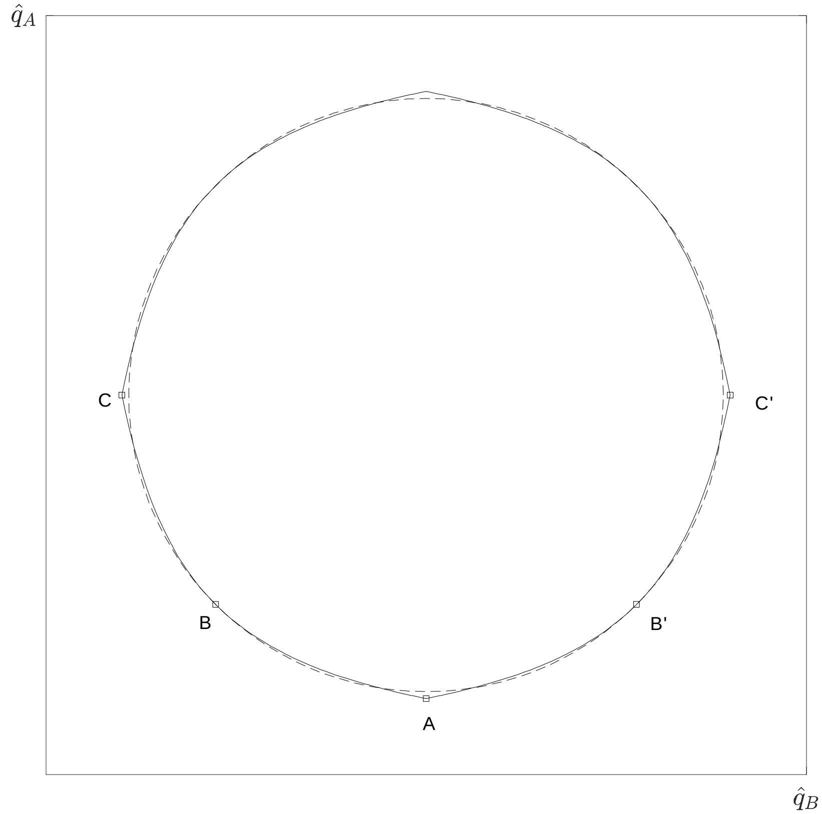

Figure 1: Core of Beliefs

The figure displays the core beliefs set, Equation (11), and the 5 cases of Lemma 2 for $d = A$ under parameter values $q_A = q_B = 100$ and $\eta^A = \ln(5)$. Point (A) is achieved in case (i), when $H_A \leq e^{\eta^A}$, which leads to extreme pessimism about a manager's own division, $\hat{q}_A << q_A$, and to reference beliefs about the other division, $\hat{q}_B = q_B$. Point (B) is achieved in case (ii), $H_A \in \left(e^{-\eta^A}, e^{\eta^A}\right)$ with $\gamma_A > 0$, which leads to moderate pessimism about both divisions, $\hat{q}_d < q_d$, $d \in \{A, B\}$. Point (B') is achieved in case (ii), $H_A \in \left(e^{-\eta^A}, e^{\eta^A}\right)$ but with $\gamma_A < 0$, which leads to moderate pessimism about a manager's own division, $\hat{q}_A < q_A$, and to optimism about the other division, $\hat{q}_B > q_B$. Point (C) is achieved in case (iii), $H_A > e^{\eta^A}$ with $\gamma_A > 0$, wherein reference beliefs are held about a manager's own division, $\hat{q}_A = q_A$, and extreme pessimism about the other division, $\hat{q}_B << q_B$. Finally, Point (C') is achieved in case (iii), $H_A > e^{\eta^A}$ but with $\gamma_A < 0$, resulting again in reference beliefs held about a manager's own division, $\hat{q}_A = q_A$, and to be extreme confidence about the other division, $\hat{q}_B >> q_B$. The dotted line represents the core of beliefs from Equation (3.12) of Chen and Epstein (2002), with $(\hat{q}_A - q_A)^2 + (\hat{q}_B - q_B)^2 \leq k_A$; this set corresponds to the relative entropy criterion for symmetric effort and zero correlation.