Uncertainty Aversion and Systemic Risk*

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Abstract

We propose a new theory of systemic risk based on Knightian uncertainty (or "ambiguity"). We show that, due to uncertainty aversion, probabilistic assessments on future asset returns are endogenous, and bad news on one asset class induces investors to hold worse expectations on other asset classes as well. This means that idiosyncratic risk can create contagion and snowball into systemic risk. Furthermore, in a Diamond and Dybvig (1983) setting, we show that, surprisingly, uncertainty aversion causes investors to be less prone to run individual banks, but runs will be systemic. In addition, we show that bank runs can be associated with stock market crashes and flight to quality. Finally, we show that increasing uncertainty makes the financial system more fragile and more prone to crises. We conclude with implications for the current public policy debate on the management of financial crisis.

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Uncertainty and waves of pessimism are the hallmark of financial crises. Financial crises and bank runs are often associated with periods of great uncertainty and sudden widespread pessimism on future returns of financial and real assets. A puzzling feature of several recent financial crises has been contagion among apparently unrelated asset classes. For example, the Asian financial crisis of 1997 spread to the Russian crisis of 1998, which eventually brought the fall of LTCM (see Allen and Gale, 1999). Negative idiosyncratic news in one asset class can also snowball into economy-wide shocks. For example, the recent crisis of 2008/2009 was triggered by negative shocks in the relatively small sub-prime mortgage market, and then rapidly spread to the general financial markets, leading to a near meltdown of the entire financial system.[1] These events raise the issue of the mechanism that triggers such contagions and put into question the very notion (and assessment) of systemic risk.

In this paper we propose a new theory of systemic risk based on uncertainty aversion. We focus on systemic risk as the possibility of a run on the (overall) banking system due contagion from one affected bank to other unaffected banks, rather than the outcome of a system-wide negative aggregate shock.[2] More generally, we study the negative spillover, due to contagion, of a negative shock affecting one asset class to other asset classes not otherwise directly affected by the shock.

Our model builds on the distinction between risk, whereby investors know the probability distribution of assets' cash flows, and Knightian uncertainty (Knight, 1921), whereby investors lack such knowledge. The distinction between the known-unknown and the unknown-unknown is relevant since investors appear to display aversion to uncertainty (or "ambiguity"), as suggested by Ellsberg (1961), as well as Keynes (1921).

We study an economy where uncertainty-averse investors hold through financial intermediaries (i.e., banks) a portfolio of risky assets. Investors perceive the distribution of the returns on the risky assets as uncertain.[3] We argue that probabilistic assessments (or *beliefs* in the sense of de

[1] Potential losses from the subprime and Alt-A mortgage markets, which in the 2007-2008 period were estimated to be in the \$100 billion to \$300 billion range, triggered losses in the world equity market in excess of \$10 trillion (see, OECD Financial Market Trends, 2007 and 2008).

[2] Note that the measurement, and the notion itself, of systemic risk is still rather controversial in the literature; see, for example, de Bandt and Hartmann (2000), Cerutti, Claessens, and McGuire (2012), and Acharya, Engle, Richardson (2012), among others, and the current discussion on macro-prudential regulation of "systemically important financial institutions."

[3] This uncertainty represents, for example, incomplete knowledge on the structure of the economy that generates asset returns, i.e., it can be viewed as model uncertainty (see Hansen and Sargent, 2008).

Finetti, 1974) held by uncertainty-averse investors on the future performance of each asset are endogenous, and depend on the composition of their portfolios. We show that this property implies that uncertainty-averse investors hold a more favorable probability assessment on the future return of an uncertain asset (i.e., are more "optimistic" on that asset) when they also hold other uncertain assets in their portfolios, a feature denoted as "uncertainty hedging." Correspondingly, bad news on one asset class induces investors to hold less favorable probability assessment on the future return of other asset classes as well and to become more "pessimistic" on those assets. Thus, a negative shock to one asset class spreads to other asset classes, creating contagion even in cases where such shocks are idiosyncratic. In this way, our paper identifies a new channel of contagion and systemic risk that is based on uncertainty aversion.

We build on the classic Diamond and Dybvig (1983) model to include two banks, each with access to a bank-specific class of risky assets (i.e., risky loans) in addition to the safe asset. Following existing literature, banks are modeled as mutual entities that maximize the welfare of their investors (i.e., depositors), who are exposed to uninsurable liquidity shocks. Banks invest in risky assets and provide investors with (partial) insurance against liquidity shocks, which exposes them to runs. Different from the more traditional "panic runs" discussed in Diamond and Dybvig (1983), in our paper we focus on fundamental runs due to the interim arrival of (idiosyncratic) bad news about a bank's expected profitability.

When investors are not uncertainty averse, there is no reason for runs to propagate from one bank to another. In contrast, due to uncertainty hedging, uncertainty-averse investors (the depositors) hold more favorable probability assessments and thus place higher value on a class of risky assets if they invest in other risky assets as well. This feature has a number of important consequences. First, it creates the possibility of contagion across banks. If a late investor withdraws early from one bank, it can now become optimal for that investor to withdraw early from the other bank as well, even if no one else runs. Thus, negative idiosyncratic shocks at any one bank can generate a deterioration of the probabilistic assessment on future returns of other banks' assets and, thus, cause runs on those banks, creating systemic risk. In this way, uncertainty aversion generates endogenous contagion and systemic risk. We also show that, interestingly, uncertainty aversion causes investors to be less prone to run individual banks, but runs will be systemic.

The distinguishing feature of our model is that uncertainty aversion is the key driver of contagion across asset classes. Specifically, in our model idiosyncratic shocks generate contagion across otherwise unrelated asset classes, and we can explain how relatively small idiosyncratic shocks can snowball into systemic risk. In contrast, absent uncertainty aversion, idiosyncratic shocks affect only the asset class directly involved by such shocks, leaving other assets classes untouched. Thus, our paper identifies a new factor of systemic risk (and contagion) that is based on investors preferences rather than on aggregate shocks that affect economy-wide fundamentals.

The second effect of uncertainty aversion is that it generates two equilibria in banks' investment decisions. When banks decide how much to invest in the risky asset, each bank is willing to make such investments if and only if the other bank invests in its risky asset as well. This implies that investors' uncertainty aversion makes investment in risky assets strategic complements, with the possibility of a second Pareto-inferior equilibrium where both banks invest in the safe asset only. This second (inefficient) equilibrium, which we denote as a "credit crunch," represents a new type of equilibrium due to coordination failure among banks, rather than among depositors as in Diamond and Dybvig (1983).

Finally, we study a more general setting with multiple heterogeneous banks and both aggregate and bank-level uncertainty. We show that increasing uncertainty makes the financial system more fragile and more prone to financial crises. Specifically, we show that for low levels of both bank-level and aggregate uncertainty, idiosyncratic shocks at a single bank generate only local runs with no contagion. At greater levels of bank level or aggregate uncertainty, idiosyncratic shocks can spread to other banks and become systemic. Finally, we show that, when aggregate uncertainty is sufficiently large, the unique equilibrium in the economy is the "credit crunch" equilibrium. In this situation, the financial system retrenches itself into a "safety mode," whereby banks refrain from lending and invest only in the safe asset, producing a "credit crunch."

We conclude our paper with a discussion of the empirical and public policy implications of our model. First and foremost, the distinguishing feature of our analysis is that financial crises can originate in one sector of the economy and then propagate through the banking system, spilling over to the stock market amidst a wave of pessimism. Conversely, our paper implies that good news in one industry can trigger additional lending to another sector, and thus result in a lending boom. We also show that, because of the externalities introduced by uncertainty aversion, banks may be exposed to a self-fulfilling (inefficient) credit crunch, whereby each individual bank is not willing to lend, even if it were (collectively) advantageous to do so.

Our paper has implications for public policy and the management of financial crises. We suggest that, when uncertainty in the economy is sufficiently low, central banks can avert runs by intervening only on the affected banks. In contrast, when the economy is exposed to greater uncertainty, bank bailouts and assets purchases by the central bank should involve not only the banks that are directly affected, but must also be extended to other banks to avoid a systemic crisis. In addition, we argue that, at high levels of uncertainty, banks may be "stuck" in a bad credit crunch equilibrium that cannot be resolved with injections of liquidity to the banking system.

Our paper is related to several stands of literature. First is the theory of bank runs based on the liquidity provision/maturity transformation role of financial intermediation originating with Diamond and Dybvig (1983). This includes Jacklin (1987), Bhattacharya and Gale (1987), Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988), and Goldstein and Pauzner (2005), among many others. Allen, Carletti, and Gale (2009) argue that aggregate volatility can induce banks to stop trading among each other, effectively generating a credit crunch.

More importantly, our paper is linked to the emerging literature on contagion and systemic risk. Allen and Gale (2000) generate contagion as the outcome of an imperfect interbank market for liquidity. Kodres and Pritsker (2002) model transmission (i.e., contagion) of idiosyncratic shocks across asset markets by investors' rebalancing their portfolios' exposures to shared macroeconomic risks among asset classes. Gârleanu, Panageas, and Yu (2014) derive contagion across assets due to limited participation and excessive portfolio rebalancing following shocks. Allen, Babus, and Carletti (2012) examine the impact of financial connections on systemic risk. Acharya, Mehran, and Thakor (2013) consider a model where regulatory forbearance induces banks to invest in correlated assets, thus creating systemic risk. Acharya and Thakor (2015) argue that, while bank leverage can be used to discipline a bank's risk-taking, it generates excessive liquidations that convey unfavorable information on the economy's fundamentals, thereby generating systemic risk. Additional papers include Freixas, Parigi, and Rochet (2000), Rochet and Vives (2004), Acharya and Yorulmazer (2008), Brusco and Castiglionesi (2007), Thakor (2015a), among many others.

Closer to our paper is Goldstein and Pauzner (2004) which argue that investors' portfolio diversification may generate systemic risk. This happens because (idiosyncratic) negative information on a bank (or, equivalently, an asset class), generates a wealth loss to investors. If investors have decreasing absolute risk aversion, this wealth loss may increase investors' risk aversion sufficiently to trigger a run on other banks that are otherwise not affected by the initial shock. Our paper differs from theirs in the fundamental mechanism that triggers contagion. Specifically, in Goldstein and Pauzner (2004) the channel of contagion is through changing the equilibrium discount rate in an economy, since the increase of investors' risk aversion affects the market risk premium. In contrast, in our model the channel of contagion is through a deterioration of investors' probability assessments on the future return of risky assets, that is, their beliefs, potentially leaving the market discount rate unaffected. Thus, the two papers complement each other, and they can jointly explain the deterioration of investor sentiment and increase of discount rates that often characterize financial crises. In addition, our paper can explain how idiosyncratic shocks of relatively small size can generate systemic runs.

Our work is also closely related to the emerging literature on uncertainty aversion in financial decision making and asset pricing.[4] Uncertainty aversion has been proposed as an alternative to Subjective Expected Utility (SEU) to describe decision making in cases where agents have only ambiguous information on probability distributions. This stream of research was motivated by a large body of work documenting important deviations from SEU and the classic Bayesian paradigm (see Etner, Jeleva, and Tallon, 2012, for an extensive survey of this literature). An important finding of this literature is that, while the degree of ambiguity aversion may vary across treatments and subjects, the presence of ambiguity aversion appears to be a robust experimental regularity. Interestingly, Chew, Ratchford, and Sagi (2013) document that ambiguity averse behavior is particularly relevant among more educated (and analytically sophisticated) subjects.

Uncertainty aversion has also been shown to be an important driver of asset pricing, providing an explanation for observed behavior that would otherwise be puzzling in the context of SEU. For example, Anderson, Ghysels, and Juergens (2009) find stronger empirical evidence for uncertainty rather than for traditional risk aversion as a driver of cross-sectional expected returns. Jeong, Kim,

[4]For a thorough literature review, see Epstein and Schneider (2008) and (2010).

and Park (2015) estimate that ambiguity aversion is economically significant and explains up to 45% of the observed equity premium. Boyarchencko (2012) shows that the sudden increase in credit spreads during the financial crisis can be explained by a surge in uncertainty faced by uncertainty-averse market participants. Dimmock et al. (2016) show that ambiguity aversion helps explain several household portfolio choice puzzles, such as low stock market participation, low foreign stock ownership, and high own-company stock ownership.[5]

Closer to our paper, Uhlig (2010) highlights the role of uncertainty aversion in a financial crisis: the presence of uncertainty-averse investors exacerbates the falls of asset prices following a negative shock in the economy. Caballero and Krishnamurthy (2008) examine a version of Diamond and Dybvig (1983) with uncertainty-averse investors. Uncertainty in their model concerns the extent of the investors' liquidity shocks (and not a bank's expected profitability, as in our model). Uncertainty aversion makes investors very pessimistic (that is, they "fear the worst") triggering a "flight-to-quality." In their model, uncertainty aversion acts as an amplification mechanism. Contagion (that is, the transmission mechanism) can happen, for example, through forced asset sales in unrelated asset markets due to investors' balance sheet constraints.[6] In our paper, uncertainty aversion itself is a new source of contagion and systemic risk.

Our paper is organized as follows. In Section 1, we outline the model. In Section 2, we develop our theory of systemic risk based on uncertainty aversion. Section 3 discusses contagion between banks and the stock market. In Section 4, we study a general model with multiple banks and both aggregate and bank-level uncertainty. Section 5 discusses the effect of increased uncertainty on fragility of the financial system. In Section 6, we discuss the empirical implications of our model and the lessons we learn for public policy and the management of financial crises. Section 7 concludes. All proofs are either in the Appendix to the paper or the Technical Online Appendix.

1 The model

We study a two-period model, with three dates, $t \in \{0, 1, 2\}$. The economy is endowed with three types of assets: a safe asset (that serves as a "storage" technology) which will be our numeraire, and

[5] See also Easley and O'Hara (2009), Bossaerts et al. (2010), Drechsler (2013), Jahan-Parvar and Liu (2014), Mele and Sangiorgi (2015), Gallant, Jahan-Parvar and Liu (2015) and Dicks and Fulghieri (2015) and (2016).

[6] See also Krishnamurthy (2010).

two classes (or types) of risky assets denominated by τ, with $\tau \in \{A, B\}$. Making an investment in a risky asset at the beginning of the first period, $t = 0$, generates at the end of the second period, $t = 2$, a random payoff denominated in terms of the safe asset. Specifically, a unit investment in the type-τ asset produces at $t = 2$ a payoff R with probability p_τ, and a payoff 0 with probability $1 - p_\tau$. A unit investment in the safe asset, which can be made either at $t = 0$ or $t = 1$, yields a unit return in the second period, so that the (net) safe rate of return is zero. We assume that returns on risky assets depend on the state of the overall economy, which provides the source of uncertainty in the model, as described below.

Our economy has two classes of players: investors and two banks. The banking system is specialized: each bank can only invest in one class of the risky asset, in addition to the safe asset. Thus, a bank of type τ can only invest in type-τ assets, for $\tau \in \{A, B\}$, at $t = 0$. This assumption captures the notion that banks in our economy are specialized lenders with a well-defined clientele. At $t = 1$, a bank has the choice of (partially) liquidating its investment in the risky technology, allowing it to recover a fraction of the initial investment. Early liquidation, however, is costly and it generates a payoff $\ell < 1$ per unit of risky asset that is liquidated at $t = 1$. Thus, liquidation of a fraction γ of the investment in risky asset τ will generate payoff $\gamma\ell$ at $t = 1$, and $(1 - \gamma)R$ with probability p_τ at $t = 2$.

The economy is populated by a continuum of investors. Each investor is endowed at $t = 0$ with \$2 in the safe asset and, as we will show later, in equilibrium will invest \$1 in bank A and \$1 in bank B. While investors have access to the storage technology (our safe asset), they can (potentially) have exposure to the risky asset only by making deposits in the banks. Following Diamond and Dybvig (1983), each investor faces at $t = 1$ a liquidity shock with probability λ.[7] Occurrence of the liquidity shock is privately observed by the investor and determines her "type." An investor hit with the liquidity shock, that is, a "short-term" investor, must consume immediately, and her utility is $u(c_1)$, with $u(0) = 0$, $u' > 0 > u''$, where c_1 is consumption at $t = 1$. An investor not impacted by the liquidity shock, that is a "long-term" investor, consumes only at $t = 2$. For analytical tractability we assume that long-term investors are risk neutral in wealth, that is, their

[7] Liquidity shocks are statistically independent across investors. Differently from Wallace (1988, 1990), and Chari (1989), among others, there is neither aggregate risk nor uncertainty on the liquidity shock.

utility is $u_2(c_2) = c_2$, where c_2 is consumption at $t = 2$.[8]

The model unfolds as follows. At the beginning, $t = 0$, banks $\tau \in \{A, B\}$ offer deposit contracts to investors. We assume that the two banks move first and simultaneously offer deposit contracts r_τ (described below) to investors, and then investors decide whether to invest their endowment as deposits at the two banks, $d_\tau \geq 0$, or to invest in the safe technology, S_a.[9] After investors make their deposits, banks decide on their investments in the safe and the risky asset. At $t = 1$, investors learn whether or not they are affected by the liquidity shock. Investors hit by a liquidity shock have no choice other than to withdraw from the bank(s) where they made a deposit and consume all their wealth. Investors not hit by a liquidity shock must decide, for each bank τ, whether to keep their deposit in the bank, $w_\tau = 0$, or to withdraw their deposits immediately, that is to "run" the bank, $w_\tau = 1$. Investors that run a bank invest the proceeds in the safe asset (i.e., the storage technology) for later consumption. At $t = 2$, cash flows from risky assets are realized and divided among investors remaining in the bank, and final consumption takes place.

An important deviation from the traditional Diamond and Dybvig (1983) framework is that we assume investors are uncertainty averse. We model uncertainty (or "ambiguity") aversion by adopting the minimum expected utility (MEU) approach promoted in Epstein and Schneider (2010).[10] In this framework, economic agents do not have a single prior on future events but, rather, they believe that the probability distribution of future events belongs to a given set M, denoted as the investor's "core beliefs set." Thus, uncertainty-averse agents maximize their MEU utility

$$\mathcal{U} = \min_{\mu \in M} E_\mu \left[u(\cdot) \right], \tag{1}$$

where μ is a probability distribution over future events, and $u(\cdot)$ is a von-Neumann Morgenstern (vNM) utility function.[11] In addition, following Epstein and Schnieder (2010), we assume that

[8] While we make the assumption that the utility for consumption at $t = 2$ is linear for analytical tractability, numerical analysis of the concave utility case yields similar results to the ones presented in our paper.

[9] Investments in risky technologies (representing loans) are available only to banks; investors have access only to the safe (storage) technology and bank deposits.

[10] MEU was originally derived by Gilboa and Schmeidler (1989). An alternative approach is "smooth ambiguity" developed by Klibanoff, Marinacci, and Mukerji (2005). In their model, agents maximize expected felicity of expected utility. Agents are uncertainty averse if the felicity function is concave. Our results follow also in that framework if the felicity function is sufficiently concave, but at the cost of requiring a substantially greater analytical complexity.

[11] In the traditional SEU framework, players have a single prior μ and maximize their expected utility $E_\mu[u(\cdot)]$.

uncertainty-averse agents are sophisticated with consistent planning. In this setting, agents are sophisticated in that they correctly anticipate their future uncertainty aversion and, thus, correctly take into account how they will behave at future dates in different states of the world.[12]

We model investor uncertainty aversion by assuming that investors are uncertain on the probability distribution of the return of the two risky assets, and we characterize the core beliefs set by using the notion of relative entropy. For given pair of (discrete) probability distributions (p, q), the *relative entropy* of p with respect to q is defined as the Kullback-Leibler divergence of p from q, and is given by

$$R(p|q) \equiv \sum_i p^i \log \frac{p^i}{q^i}. \tag{2}$$

The core beliefs set for the uncertainty-averse investors in our economy is then given by

$$M \equiv \{p : R(p|q) \leq \eta\}, \tag{3}$$

where p is the joint distribution of the returns on the two risky assets and q is a certain, exogenously given "reference" probability distribution of the return on the risky assets. Thus, the core beliefs set M is the set of distributions p with a divergence not greater than η with respect to the reference distribution q. The parameter η can be interpreted as representing the extent of uncertainty that is present in the economy.[13] Note that, if the return distributions on the two risky assets are independent (as we assume in our paper), from (2) and (3) it can immediately be seen that $R(p|q) = R(p_A|q_A) + R(p_B|q_B)$ and thus that

$$M = \{p : R(p_A|q_A) + R(p_B|q_B) \leq \eta\}. \tag{4}$$

Expression 4 has the appealing interpretation that, for given total uncertainty (i.e., entropy) in the economy, η, an increase in the uncertainty on the return distribution of one asset, $R(p_\tau|q_\tau)$, requires a corresponding decrease of uncertainty on the return distribution of the other asset, $R(p_{\tau'}|q_{\tau'})$, $\tau \neq \tau'$. It is immediate to verify the following property of the core beliefs set M.

[12] Siniscalchi (2011) describes this framework as preferences over trees.

[13] As in Epstein and Schneider (2010), Hansen and Sargent (2005), (2007), and (2008), relative entropy can also be interpreted as characterizing the extent of "misspecification error" that affects investors.

Lemma 1 *Let $\eta < \underline{\eta}(q)$, defined in the appendix. The core beliefs set M is a strictly convex set with smooth boundary.*

Note that Lemma 1 is an implication of the fact that relative entropy $R(p|q)$ is a strictly convex function of p.[14] Lemma 1 also implies that, for uncertainty-averse agents with positive endowment of the underlying risky assets, the relevant part of the core beliefs set M is a smooth, decreasing and convex function. This property is an implication of the fact that uncertainty-averse agents solving problem (1), will select their probability assessments that lie in the "lower-left" boundary of the core beliefs set M. See Figure 1 on page 42.

We model the core beliefs set as follows. We assume that the success probability of an asset of type-τ depends on the value of an underlying parameter θ_τ, and is denoted by $p_\tau(\theta_\tau)$, with $\theta_\tau \in [\theta_L, \theta_H] \subseteq [\theta_m, \theta_M]$, $\theta_M - \theta_H = \theta_L - \theta_m$. For analytical tractability, we assume that $p_\tau(\theta) = e^{\theta_\tau - \theta_M}$, with $\tau \in \{A, B\}$. Uncertainty-averse agents treat the vector $\vec{\theta} \equiv (\theta_A, \theta_B)$ as ambiguous and assess that $\vec{\theta} \in C \subset \{(\theta_A, \theta_B) : (\theta_A, \theta_B) \in [\theta_L, \theta_H]^2\}$. We interpret the parameter combination $\vec{\theta}$ as describing the state of the economy at $t = 2$ and we denote C as the set of "core beliefs" of our uncertainty-averse investors.

In light of Lemma 1 and subsequent discussion, we assume that for $\vec{\theta} \in C$ we have that $(\theta_A + \theta_B)/2 = \theta_T$, where $\theta_T \equiv (\theta_H + \theta_L)/2$; see Figure 1 for an illustration. Thus, in this specification, a greater value of the parameter θ_A increases the success probability of type A assets, but it comes at the expense of a decrease of the success probability of type B assets. Thus, a greater value of θ_A is more "favorable" for asset A and more "unfavorable" to asset B. From 4, this property can be interpreted as modeling the situation where, for given total uncertainty in the economy, more uncertainty on the probability distribution of the return on one asset is balanced by less uncertainty on the probability distribution of the other asset. Note also that, for a given value of the parameter combination $\vec{\theta}$, the probabilities distributions $p_\tau(\theta_\tau)$, $\tau \in \{A, B\}$, are independent. This means that the returns on the risky assets are uncorrelated.[15]

We will at times benchmark the behavior of uncertainty-averse agents with the behavior of an uncertainty-neutral SEU agent, and we will assume that uncertainty-neutral investors has $\theta_L = \theta_H$,

[14] For a general discussion, see Theorem 2.5.3 and 2.7.2 of Cover and Thomas (2006).

[15] Our model can easily be extended to the case where, given θ, the realization of the asset payoffs at the end of the period are correlated.

so that she assesses $\theta_\tau = \theta_T$. This assumption guarantees that the uncertainty-neutral investor has the same probability assessment on the return on the two assets as a well-diversified uncertainty-averse investor (and thus there is no "hard-wired" difference between the to type of investors). We will also assume throughout that $e^{\theta_T - \theta_M} R > 1$. This inequality implies that the expected profits from risky assets are sufficiently large to make an uncertainty-neutral investor willing to invest in such assets. Later, we will also show that this implies a well-diversified uncertainty-averse investor is willing to invest in the uncertain assets.

1.1 Deposit contracts

We assume that banks are organized as "mutual" financial institutions, such as mutual saving banks or credit unions, and maximize the welfare of their depositors. Thus, at the beginning of the first period, $t = 0$, banks offer investors deposit contracts that maximize their lifetime welfare.[16] Because banks can make risky investments, departing from Diamond and Dybvig (1983), the payoff from deposit contracts depends both on the date of withdrawal and the realization of the investment in the risky asset, if a bank makes such investment. Thus, a deposit contract offered by bank τ is a triplet $r_\tau \equiv \left\{ r_{1\tau}, r_{2\tau}^l, r_{2\tau}^h \right\}$ describing the time- and state-dependent payoff to a depositor per unit of deposits, as follows.[17] Given a unit deposit at time $t = 0$, investors who withdraw at $t = 1$ receive safe payoff $r_{1\tau} \geq 0$. Investors keeping their deposits at the bank until $t = 2$, receive a payoff that that can be composed by two parts: first, that they receive a safe payoff $r_{2\tau}^l \geq 0$ which is independent of the realization of the risky asset, plus they may receive a second payoff $r_{2\tau}^h \geq 0$ which is paid to the investor only if the risky asset has generated the high return R. There is no government insurance guarantee for deposits.

Given a deposit contract $r_\tau \equiv \left\{ r_{1\tau}, r_{2\tau}^l, r_{2\tau}^h \right\}$, an investor depositing $d_\tau \geq 0$ dollars at bank τ receives a total payoff (and consumption) from holding her deposits in the two banks, as follows. In absence of runs, investors hit with the liquidity shock withdraw early and receive from each bank

[16] Alternatively, we could assume that the banking sector is open to free entry, whereby a type-τ bank is exposed to potential competition from banks of the same type. Zero-profit condition ensures that at the beginning of the period, $t = 0$, a type-τ bank offers investors a deposit contract that maximize their lifetime welfare. Note that, in this case, to be able to raise deposits from investors, a bank must be able to commit, at the time deposits are made by investors, to their asset allocation between the safe and risky assets.

[17] Because banks maximize investors' ex-ante utility, optimal consumption allocations can be implemented with linear deposit contracts WLOG (see the proofs of Theorems 1 and 2).

a payoff equal to $r_{1\tau}d_\tau$ and, thus, their consumption is equal to $c_1 = S_a + r_{1A}d_A + r_{1B}d_B$, where $S_a \geq 0$ is an investor's investment in the safe asset. Investors not hit with the liquidity shock and holding their initial deposits with both banks have consumption which will depend on the realized return on each of the risky assets, with $E(c_2) = S_a + (r^l_{2A} + p_A r^h_{2A})d_A + (r^l_{2B} + p_B r^h_{2B})d_B$.

We will initially focus on equilibria with no runs. To simplify the exposition, let U_0 be the value function of investors at $t = 0$, and let U_1 be the value function of late investors in the case of no runs.[18] Thus,

$$U_0 \equiv \lambda u\left(S_a + r_{1A}d_A + r_{1B}d_B\right) + (1-\lambda)\, U_1\left(\vec{\theta}_1\right), \tag{5}$$

$$U_1\left(\vec{\theta}_1\right) \equiv S_a + \left(r^l_{2A} + e^{\theta_A - \theta_M} r^h_{2A}\right) d_A + \left(r^l_{2B} + e^{\theta_B - \theta_M} r^h_{2B}\right) d_B, \tag{6}$$

where $\vec{\theta}_1$ is their belief about the state of the economy, which we derive next.

1.2 Endogenous beliefs

An important implication of uncertainty aversion is that the investor assessments on the parameter combination $\vec{\theta}$ depends on their overall exposure to risk and, thus, on the structure of their portfolios. This means that the probability assessment (i.e., the "beliefs") held by an uncertainty-averse investor on the state of the economy (that is, the parameter combination $\vec{\theta}$) are endogenous, and depend on the agent's overall exposure to the risk factors of the economy.

Endogeneity of beliefs is the outcome of the fact that the minimization operator in (1), which determines the probability assessment held by an investor, in general depends on the composition of the investor's overall portfolio. It is useful to note that this property, which plays a critical role in our paper, implies that uncertainty-averse agents are more willing to hold uncertain assets if they can hold such assets in a portfolio rather than in isolation. This happens because, by holding uncertain assets in a portfolio, investors can lower their overall exposure to the sources of uncertainty in the economy, a property that we will refer to as *uncertainty hedging*.[19]

The effect of uncertainty hedging in our model is that investors hold more favorable probability

[18] The payoff to early and late investors in the case of runs are displayed in the Appendix. The corresponding expressions for U_0 and $U_1\left(\vec{\theta}_1\right)$ in the case of runs on one, or both, banks are available in the Appendix.

[19] This property can be loosely interpreted as the analogue for MEU investors of the more traditional "benefits of diversification" displayed by SEU preferences. The property may be seen immediately by noting that, given two random variables, y_k, with distributions $\mu_k \in \mathcal{M}$, $k \in \{1, 2\}$, which are ambiguous to agents, by the property of the

assessments on the future return of the risky assets held by the two banks when they make deposits in both banks, rather than when they make deposits in only one bank. This property may be seen as follows. Long-term investors' ultimate exposure to the sources of uncertainty in the economy depends on the initial deposits made at each bank, d_τ, the investors' decision on whether or not to keep these deposits at each bank, w_τ, and the deposit contracts offered by each bank, r_τ. Because of uncertainty aversion, from (6), the investor's assessment at $t = 1$ on the state of the economy is the solution to the minimization problem:

$$\vec{\theta}_1^a \equiv \arg\min_{\vec{\theta}_1 \in C} U_1\left(\vec{\theta}_1\right), \tag{7}$$

and is characterized in the following Lemma.

Lemma 2 *Let*

$$\check{\theta}_\tau \equiv \theta_T + \frac{1}{2} \ln \frac{r_{2\tau'}^h d_{\tau'}\left(1 - w_{\tau'}\right)}{r_{2\tau}^h d_\tau \left(1 - w_\tau\right)}. \tag{8}$$

The assessment held at $t = 1$ by an uncertainty-averse agent on the state of the economy is $\vec{\theta}_1^a = (\theta_A^a, \theta_B^a)$, where

$$\theta_\tau^a = \begin{cases} \theta_L & \check{\theta}_\tau \leq \theta_L \\ \check{\theta}_\tau & \check{\theta}_\tau \in (\theta_L, \theta_H) \\ \theta_H & \check{\theta}_\tau \geq \theta_H \end{cases} \quad \textit{for} \quad \tau \in \{A, B\}. \tag{9}$$

Lemma 2 shows that investor assessments at $t = 1$ on the expected future profitability of the two banks, as it is affected by the state of the economy (captured by the parameter θ_τ^a), depends critically on the composition of their overall portfolio. We will denote $\vec{\theta}_1^a$ as characterizing investor "beliefs." We will say that the investor has *interior beliefs* when $\theta_\tau^a \in (\theta_L, \theta_H)$ for $\tau \in \{A, B\}$. Otherwise, we will say that the investor holds *corner beliefs.* The following lemma can immediately be verified.

Lemma 3 *Holding deposits d_τ in bank τ constant, a decrease in the investor's deposit in type-τ'*

minimum operator we have, for $q \in [0, 1]$, that

$$q \min_{\mu \in \mathcal{M}} E_\mu \left[u\left(y_1\right)\right] + (1 - q) \min_{\mu \in \mathcal{M}} E_\mu \left[u\left(y_2\right)\right] \leq \min_{\mu \in \mathcal{M}} \{q E_\mu \left[u\left(y_1\right)\right] + (1 - q) E_\mu \left[u\left(y_2\right)\right]\}.$$

bank, $d_{2\tau'}$, with $\tau' \neq \tau$, leads the investor to decrease her assessment of the success probability of the assets held by a type-τ bank, that is, to lower θ^a_τ.

Lemma 3 shows that when an investor has a relatively greater proportion of her portfolio deposited in a bank (determined, for example, by a decrease in an investor's deposit in the other bank), she will be relatively more concerned about the priors that are less favorable to assets held by that bank. Thus, the investor will give more weight to priors that are less favorable to that bank. This implies that, in the optimization problem (7), the investor will select values of the parameter θ which are less favorable to bank τ, leading to lower values of θ^a_τ and, thus, of the success probability $p_\tau(\theta^a_\tau)$. In other words, the investor will be more "pessimistic" about the return on that asset. In turn, the investor will hold priors more favorable to the other asset and, thus, will become correspondingly more "optimistic" with respect to the other asset.

If an uncertainty-averse investor withdraws her deposit from one bank, $w_{\tau'} = 1$, and holds deposits only at the other bank, $w_\tau = 0$, she will have a probability assessment on the return on the assets held by bank τ determined by the worse-case scenario for that bank, with $\theta^a_\tau = \theta_L$. Similarly, if at $t = 0$ an investor deposits her endowment only in one bank, she will have beliefs on the return on the assets held by the bank that are determined by the worse-case scenario: $\theta^a_\tau = \theta_L$.

Lemma 2 will play a crucial role in our analysis. Specifically, it means that uncertainty aversion creates complementarities between investments in different asset classes, so that investors are more optimistic, and thus value more, one class of risky assets if they can also invest in other risky assets. Such portfolio complementarity for investors, in turn, induces a strategic complementarity among banks, resulting in multiple equilibria. It also implies that (idiosyncratic) bad news about a bank, which will induce a run on that bank, will make investors more pessimistic about the other bank's profitability, possibly triggering a run also on that other bank. In this way, the presence of uncertainty aversion creates contagion, and thus systemic risk.

1.3 Optimal deposit contracts and investment policy

We now examine the optimal deposit contracts offered by banks and their optimal investment policy in the safe and the risky technology. In particular, bank τ sets the optimal deposit contract, r_τ, offered to investors and the levels of investment in the safe and risky technologies, $S_\tau \geq 0$ and

$K_\tau \geq 0$, per unit of deposits d_τ, given the optimal contract and investment policy adopted by the rival bank τ', to maximize investors' ex-ante utility

$$\max_{\{r_\tau, S_\tau, K_\tau\}} \quad U_0 \equiv \lambda u(c_1) + (1-\lambda) U_1 \left(\vec{\theta}_1^a\right) \tag{10}$$

subject to the following constraints. Note that, while problem (10) characterizes the level of investment in the safe and risky asset that is ex-ante optimal, we will show in the Appendix that these investment levels remains optimal after a bank receives the deposits from investors.

Because liquidity shocks are privately observable only to investors at the interim date, $t = 1$, deposit contracts offered by a bank must satisfy the appropriate incentive compatibility constraints. Early investors must consume immediately, since they gain no utility from $t = 2$ consumption, giving

$$c_1 = S_a + r_{1A} d_A + r_{1B} d_B. \tag{11}$$

Late investors, in contrast, may pretend to be early investors and withdraw their deposits from either (or both) banks and invest in the safe technology for later consumption. Thus, to prevent runs on one (or both) banks, deposit contracts must satisfy

$$U_1 \left(\vec{\theta}_1^a\right) \geq S_a + r_{1\tau} d_\tau + r_{1\tau'} d_{\tau'}, \tag{12}$$

$$U_1 \left(\vec{\theta}_1^a\right) \geq S_a + r_{1\tau} d_\tau + (r_{2\tau'}^l + e^{\theta_L - \theta_M} r_{2\tau'}^h) d_{\tau'}, \tag{13}$$

$$U_1 \left(\vec{\theta}_1^a\right) \geq S_a + r_{1\tau'} d_{\tau'} + (r_{2\tau}^l + e^{\theta_L - \theta_M} r_{2\tau}^h) d_\tau, \tag{14}$$

where (12) ensures that late investors prefer keeping their deposits in both banks rather than running on both of them, (13) ensures that late investors prefer not to run bank τ, while keeping their deposits in bank τ', and (14) ensures that late investors prefer not to run bank τ', while keeping their deposits in bank τ. Note that the incentive compatibility constraint (13) reflects the fact that, if a long term investor runs bank τ and not bank τ', she will have a portfolio that is exposed only to the risk of type-τ' assets only. This implies that she will be concerned only with the states of the economy that are least favorable to risky asset τ' and, thus, will set $\theta_{\tau'}^a = \theta_L$. Similarly, if the long-term investor runs bank τ', the investor will be concerned only with the states of the economy that are least favorable to risky asset τ and, thus, will set $\theta^a_\tau = \theta_L$, leading to (14). Banks correctly anticipate investors' probability assessments $\vec{\theta}^a_1$ (i.e., their beliefs) at $t = 1$:

$$\vec{\theta}^a_1 = \arg\min_{\vec{\theta}_1 \in C} U_1\left(\vec{\theta}_1\right), \tag{15}$$

$$U_1\left(\vec{\theta}_1\right) \equiv S_a + (r^l_{2\tau} + e^{\theta_\tau - \theta_M} r^h_{2\tau}) d_\tau + (r^l_{2\tau'} + e^{\theta_{\tau'} - \theta_M} r^h_{2\tau'}) d_{\tau'}. \tag{16}$$

Finally, the optimal deposit contract satisfies a bank's budget constraints at time $t = 0, 1, 2$ regarding investments in the safe and risky technology, and promised payoffs in the deposit contract:

$$1 \geq S_\tau + K_\tau \tag{17}$$

$$S_\tau \geq \lambda r_{1\tau} + (1-\lambda) r^l_{2\tau}, \tag{18}$$

$$K_\tau R \geq (1-\lambda) r^h_{2\tau}. \tag{19}$$

Note that, if a deposit contract, r_τ, offered to investors by a bank does not satisfy the incentive-compatibility and feasibility constraints (12) - (19), investors will anticipate a run and will not be willing to make any deposit in the bank. We will make the following additional assumptions:

Conditions A_0 (Regularity conditions):

$$u'(2) > e^{\theta_T - \theta_M} R > u'\left(2 \frac{e^{\theta_T - \theta_H} R}{\lambda e^{\theta_T - \theta_H} R + (1-\lambda)}\right). \tag{20}$$

The first inequality ensures that the optimal deposit contract offered by banks to uncertainty-neutral investors provides (partial) insurance against liquidity shocks, while the second inequality ensures that the optimal deposit contracts satisfy the incentive compatibility constraint (12) with strict inequality, that is, that the constraint is not binding in the optimal contract.[20]

Condition A_1 (Contagion):

$$e^{\theta_L - \theta_M} R < 1.$$

This inequality implies that there are priors in the core beliefs set such that an investor assessing

[20] Note that the regularity conditions A_0 have the same role as the assumptions in Diamond and Dybvig (1983) that investors have a coefficient of RRA greater than 1 and that $\rho R > 1$, which together ensure that in, in their model, the optimal deposit contract $\{r^*_1; r^*_2\}$, satisfies $1 < r^*_1 < r^*_2 < R$.

cash flows with such priors is not willing to make a unique deposit in a bank of type τ, for $\tau \in \{A, B\}$. As will become apparent below, A_1 implies that, while an uncertainty-averse investor would be willing to make deposits in both types of banks, she may not be willing to keep her deposit in a bank of one type only. This features creates the possibility of systemic runs.

1.4 Equilibrium banking

We now characterize the equilibria in the basic game. We will use the notion of subgame-perfect Nash Equilibrium.

Definition 1 *A subgame-perfect Nash Equilibrium in our economy is a strategy combination* $\{r_\tau^*, d_\tau^*, S_a^*, S_\tau^*, K_\tau^*, w_\tau^*\}$ *such that (i) each bank* $\tau \in \{A, B\}$ *selects the initial deposit contract offered to investors,* r_τ^**, and its investment policy in the safe,* S_τ^**, and risky technology,* K_τ^**, that maximizes investors' ex-ante utility,* U_0*, subject to (11) - (19), and given the other bank's and the investors' optimal strategies; (ii) an allocation at* $t = 0$ *of deposits by investors between the storage technology,* $S_a \geq 0$*, and two banks,* $d_\tau^* \geq 0$*, with* $S_a + d_A^* + d_B^* \leq 2$*, given the deposit contacts* r_τ^* *offered by the two banks, that maximizes their ex-ante utility,* U_0*, and a withdrawal policy for late investors,* w_τ^**, that maximizes their continuation utility,* U_1*.*

As a benchmark we consider first the case in which agents are uncertainty-neutral, as follows (recall that $\theta_L = \theta_H = \theta_T$ for uncertainty-neutral investors).

Theorem 1 *If investors are uncertainty neutral, there is an equilibrium deposit contract* $r_\tau^{\rho *} \equiv \left\{r_{1\tau}^{\rho *}, r_{2\tau}^{l\rho *}, r_{2\tau}^{h\rho *}\right\}$ *such that*

$$d_\tau^{\rho *} = 1,\ r_{2\tau}^{l\rho *} = 0,\ \text{and } 1 < r_{1\tau}^{\rho *} < e^{\theta_T - \theta_M} r_{2\tau}^{h\rho *},\ \text{for } \tau \in \{A, B\}, \tag{21}$$

that is, banks provide partial insurance against liquidity shocks, investors invest all their endowment equally in both banks, $d_\tau^* = 1$*, and do not run,* $w_\tau^* = 0$*.*

Theorem 1 shows that, as in Diamond and Dybvig (1983), a symmetric equilibrium with $r_{1A}^{\rho *} = r_{1B}^{\rho *}$ and $r_{2A}^{h\rho *} = r_{2B}^{h\rho *}$ always exists, whereby banks provide investors with partial insurance against liquidity shocks: $1 < r_{1\tau}^{\rho *} < e^{\theta_T - \theta_M} r_{2\tau}^{h\rho *}$. In addition, just like in Diamond and Dybvig (1983), insurance provision by banks implies that, in equilibrium, banks are illiquid and, thus, potentially exposed to runs. It is, however, important to note that, although runs do not occur in equilibrium, if a run on one bank did occur, it would not trigger a run on the other bank. Thus, runs would not be systemic: a run on one bank would not necessarily induce a run on the other bank, so the banking system is not fragile.[21]

These properties change dramatically when investors are uncertainty averse. From Lemma 2 we know that, because of uncertainty aversion, the investors' assessment on the future state of the economy and, thus, on banks' expected solvency, depends on their overall risk exposure. In this way, uncertainty aversion creates a direct link between investor's desired holding in each asset class, making asset holdings effectively complements.

The strategic complementarity due to uncertainty aversion generates the possibility of multiple equilibria and systemic runs. There are two types of equilibria when investors are uncertainty averse. The first type of equilibrium has the same properties as the one in which investors are uncertainty neutral, as described in Theorem 1. In this equilibrium, banks invest in the risky assets, offer partial insurance to investors, are illiquid and exposed to runs. We will denote this equilibrium as the "risky" equilibrium. We interpret this equilibrium as one in which banks carry out their normal lending activity.

In the second equilibrium, banks invest only in the safe asset, making the banking system effectively immune to runs, an equilibrium we will denote as the "safe" equilibrium. In this second "safe" equilibrium, banks refrain from investing in the (potentially) more profitable risky assets and, rather, invest only in the safe asset. We interpret this equilibrium as a "credit crunch," where banks invest only in safe assets and refrain from lending.

Theorem 2 *If investors are uncertainty averse and A_1 holds, there are both a "risky" equilibrium, where the optimal deposit contract is again $r_\tau^{\rho*}$ characterized in (21), and a "safe" ("credit crunch") equilibrium, in which both banks invest only in the safe asset and offer a safe deposit contract, $r_\tau^{\sigma*} = \left\{r_{1\tau}^{\sigma*}, r_{2\tau}^{l\sigma*}, r_{2\tau}^{h\sigma*}\right\}$, and no insurance against liquidity risk: $r_{1\tau}^{\sigma*} = r_{2\tau}^{l\sigma*} = 1$ and $r_{2\tau}^{h\sigma*} = 0$, for*

[21] For completeness, note that there are also "virtual run" equilibria, whereby if investors expect a run at $t = 1$, in either or both banks, $w_\tau^* = 1$, $\tau \in \{A, B\}$, they do not make any deposit at the affected bank at the initial period, $t = 0$. Similarly, under MEU, if investors expect a run at any one of the two banks, they will make no deposits at any bank. Since these nonparticipation, or "autarky," equilibria are not interesting, we will ignore them in the rest of the paper. See Allen and Gale (2007) for a general discussion.

$\tau \in \{A, B\}$. Again, investors invest equally in both banks, $d^*_\tau = 1$. Furthermore: (i) The "risky" equilibrium Pareto dominates the "safe" equilibrium; (ii) banks are not exposed to runs in the "safe" equilibrium, but they are in the "risky" equilibrium.

Theorem 2 shows that the presence of uncertainty aversion has the effect of creating a second equilibrium, in addition to the one prevailing in an economy populated by SEU agents. Specifically, in addition to the equilibrium where banks invest in risky technology and offer (partial) insurance against liquidity shocks that prevails when investors are uncertainty neutral, there is also a credit crunch equilibrium in which banks invest only in the safe asset. The credit crunch equilibrium is inefficient: it is Pareto dominated by the risky equilibrium where both banks invest in their respective risky assets.

Existence of the credit crunch equilibrium depends critically on the fact that an uncertainty-averse investor is willing to deposit funds in one type of bank and, thus, be exposed to one type of risk, only if she also can invest in the other bank and, thus, be exposed to the other source of risk as well. This implies that if one bank offers only the safe deposit contract, the other bank will only offer the safe deposit contract as well. This happens because, if to the contrary a single bank offers to investors a risky deposit contract, from Lemma 2 this offer will be met by investors with beliefs that correspond to the worst-case scenario for that bank. In this case, Condition A_1 implies that making a deposit at that bank is perceived by investors as a negative NPV investment. In other words, a bank offering a risky deposit contract is perceived by investors as insolvent (in expected value) and they will refuse to make any deposit in that bank.

The strategic externality in the investment policy of banks is due to the fact that uncertainty-averse investors are willing to make a deposit in one bank only if they have the opportunity to invest in the other bank as well. This externality creates the potential of a "coordination failure" among banks and multiple equilibria. This coordination failure among banks and the possibility of credit crunch equilibria is new in the literature, and it complements the more traditional coordination failure among investors that can generate "panic runs" as in Diamond and Dybvig (1983).

Selection between the risky and the credit crunch equilibrium is an open question. Pareto optimality of the risky equilibrium suggests that banks may spontaneously focus on such equilibrium. However, we would like to recognize the possibility that, in time of financial crises, banks may shift to the credit crunch equilibrium. A shift from the risky to the credit crunch equilibrium may occur, for example, as a consequence of an external event, such as the release of bad news on the economy that acts as coordination device. In Section 5, we will show that there are circumstances in which only the credit crunch equilibrium exists.

A second important effect of uncertainty aversion is that, although runs do not occur in equilibrium of the basic model, if a run on one bank does occur, it causes also a run on the other bank. This happens because a run at $t = 1$ by long-term investors on one bank shifts the composition of their portfolios in favor of the other bank. From Lemma 2, this change of investors' portfolio composition causes investors to become more pessimistic on the return on the asset of the bank whose deposits are still in their portfolios, triggering a run on that bank as well. Thus, uncertainty aversion creates the possibility of systemic risk, which we will examine next.

2 Uncertainty aversion and systemic risk

Existing literature has examined two distinct categories of runs in a bank economy: panic runs and fundamental runs. Panic runs, as first discussed in Diamond and Dybvig (1983), occur when investors run a bank, even though the bank would still be solvent if they did not run, and investors would prefer the outcome of no one running. Panic runs are essentially due to a coordination failure among investors in a situation where a bank would otherwise be solvent. A fundamental run occurs when there is a shock to economic fundamentals large enough so that it ceases to be optimal for a long-term investor to remain invested in the bank, even if everyone else stays in the bank.

A further important distinction is between runs that involve only one bank and, thus, are "local" and runs that involve a large number of banks and, thus, are "systemic." Systemic runs may be the outcome of a system-wide negative shock that affects the aggregate economy. In contrast, and of interest here, are runs that originate from a shock to a small part of the banking sector and then propagate by contagion from affected banks to nonaffected ones.

In our paper we focus on systemic runs caused by contagion. From Theorem 1 and Theorem 2 we know that, in our economy, although runs do not occur in equilibrium, banks are always exposed to the possibility of a run in a risky equilibrium. However, when investors are uncertainty neutral, runs may not necessarily spread from one bank to the other. In contrast, if investors are uncertainty averse, contagion across banks may occur and runs can become systemic.

To model the possibility of equilibrium runs, similar to Goldstein and Pauzner (2005), we now assume that, at $t = 1$, investors receive public signals, s_τ, $\tau \in \{A, B\}$, that are informative on the magnitude of the payoff given success from the risky assets at time $t = 2$. Specifically, we assume that $R_\tau = s_\tau R$, with $s_\tau \in \{\phi, 1\}$ and $\phi < 1$. We also assume that with probability $\varepsilon > 0$ investors observe "bad news" about type τ assets only, $s_\tau = \phi$ and $s_{\tau' \neq \tau} = 1$, for $\tau \in \{A, B\}$, while with probability Δ, investors observe "bad news" about both type A and type B assets, $s_\tau = s_{\tau' \neq \tau} = \phi$, and with probability $1-2\varepsilon-\Delta$, investors learn that both asset classes are unaffected, $s_\tau = s_{\tau' \neq \tau} = 1$. Because "bad news" about both banks generate the expected and arguably uninteresting outcome of fundamental systemic runs, we set $\Delta = 0$. For tractability, we now assume that investors' utility function, u, is piece-wise affine. Specifically,

$$u(c) = \begin{cases} \psi c & c \leq \tilde{c} \\ \psi \tilde{c} + (c - \tilde{c}) & c > \tilde{c} \end{cases} \tag{22}$$

where $\psi > e^{\theta_T - \theta_M} R > 1$, and $\tilde{c} \in \left(2, 2\frac{e^{\theta_T - \theta_M} R}{\lambda e^{\theta_T - \theta_M} + (1-\lambda) R}\right)$. This utility function captures the notion that early investors value lower consumption levels, up to $\tilde{c}$, relatively more than larger consumption, and that they value consumption more than late investors, preserving the value of insurance against the liquidity shock.

The payoff to early and late investors in the case of runs on one or both banks are determined as follows. If late investors run a bank, say bank τ, early and late investors receive a payment which depends on the proportion of investors that withdraw their deposits early, $n_\tau \geq \lambda$, as follows . If the number of investors demanding early withdrawal is sufficiently low, $n_\tau \leq (S_\tau + \ell K_\tau)/r_\tau^1$, banks will honor the promised payment r_τ^1 out of their investment in the safe asset, S_τ, and possibly by liquidating their investment in the risky asset, K_τ. In contrast, if the number of investors demanding early withdrawal is large, $n_\tau > (S_\tau + \ell K_\tau)/r_\tau^1$, banks will not have sufficient funds to pay all investors the promised amount r_τ^1. In this case, we assume that banks follow a sequential service constraint, which implies that the first $(S_\tau + \ell K_\tau)/\left(\eta_\tau r_\tau^1\right)$ investors that withdraw their deposits at $t = 1$ can obtain the full promised payment r_τ^1, while the remaining investors that withdraw early receive 0.[22]

Correspondingly, late investors that do not withdraw early, receive a random payoff which is determined as follows. If $n_\tau \leq S_\tau / r_\tau^1$, late investors receive a safe payment of $(S_\tau - n_\tau r_\tau^1)/(1 - n_\tau)$, plus the promised payment $r_{2\tau}^h$, if the risky asset has the high return R. If $S_\tau / r_\tau^1 < n_\tau \leq (S_\tau + \ell K_\tau)/r_\tau^1$, banks will have liquidated at $t = 1$ their entire holdings of the safe asset and also have (partially) liquidated the risky asset as well to satisfy the demands investors withdrawing deposits at the time. Thus, in this case, late investors will receive a payoff of $\left(K_\tau - \frac{n_\tau r_\tau^1 - S_\tau}{\ell}\right) \frac{R}{1-n_\tau}$ only if the risky asset has a high return R. Finally, if $n_\tau > (S_\tau + \ell K_\tau)/r_\tau^1$, banks will have also liquidated their investment in the risky asset at $t = 1$ to satisfy the demand of the early investors, and late investors will receive zero payoff with probability one.

We now establish the existence of equilibrium systemic runs due to uncertainty aversion. We proceed in two steps. We start the analysis by establishing, in the following lemma, the possibility of systemic runs under uncertainty aversion for a given and arbitrary deposit contract $r_\tau \equiv \left\{r_{1\tau}, r_{2\tau}^l, r_{2\tau}^h\right\}$, $\tau \in \{A, B\}$. We will then characterize the optimal deposit contracts and the overall equilibrium in the next theorem.

Lemma 4 *Let $r_\tau \equiv \left\{r_{1\tau}, r_{2\tau}^l, r_{2\tau}^h\right\}$, $\tau \in \{A, B\}$ be symmetric deposit contracts with $r_{1\tau} > 1$, $r_{2\tau}^l = 0$, $r_{2\tau}^h > 0$ (i.e., risky deposit contracts) and $d_A = d_B$. If investors are uncertainty neutral, they will run bank τ following bad news about type τ assets if $r_{1\tau} > \phi p_\tau(\theta_T) r_{2\tau}^h$, but investors will not run bank $\tau' = \tau$. If investors are uncertainty averse, they will run both banks if $r_{1\tau} > \phi^{\frac{1}{2}} p_\tau(\theta_T) r_{2\tau}^h$.*

Lemma 4 uncovers a new source of systemic risk that is due to uncertainty aversion, and provides one of the key results of our paper. The lemma shows that, in the presence of uncertainty-averse investors, bad news at one bank, say bank A, while it generates a fundamental run on that bank, also induces investors to run on the other bank, bank B, even in the absence of bad news at the latter bank. Thus, bad news on one bank can create a systemic run: idiosyncratic risk can indeed generate systemic risk.

[22] We assume that each investor's position "in line" at a bank to make an early withdraw is random, and that all investors have an equal probability of receiving the positive payoff.

The mechanism behind the systemic risk described in Lemma 4 is the uncertainty hedging motive due to uncertainty aversion. As shown in Lemma 2, investor assessments of the success probability of a risky asset depends on their overall portfolio. In particular, an uncertainty-averse investor is willing to make a deposit in one bank, and thus to be exposed to the risky asset held by that bank, provided that she makes a deposit in the other bank as well, and thus be exposed also to the other risky asset. This implies that, if the investor learns bad news about one risky-asset class, say $\tau = A$, inducing a run on bank A, the investor's portfolio will become overly exposed to the other risky asset class, $\tau = B$. From Lemma 3, we know that the resulting portfolio imbalance causes a shift in the investor's assessment against the other asset class, B, making the investor relatively more pessimistic about risky asset B. Thus, a run on bank B may happen even if that bank was not affected by bad news. Thus, bad news about bank A spills over to bank B causing contagion and, thus, systemic risk. Note that this source of contagion and systemic risk is entirely driven by uncertainty aversion and is novel in the literature. It will be denoted as "uncertainty-based" systemic risk, which generates "uncertainty-based" systemic runs.

Lemma 4 describes investors' behavior in response to negative shocks, given an arbitrary deposit contract. Banks, however, offer ex-ante optimal deposit contracts that anticipate such behavior. The following theorem determines the ex-ante optimal deposit contracts offered by banks, incorporating the expectation of equilibrium runs after bad news.

Theorem 3 *Let early investors have piecewise affine utility as in (22) and ε be small enough.*
(i) If investors are uncertainty-neutral SEU agents, the equilibrium is a risky equilibrium where banks invest in the risky technology and provide insurance against the liquidity shock by offering the deposit contract:

$$r_{1\tau}^{\rho*} = \frac{1}{2}\tilde{c},\ r_{2\tau}^{l\rho*} = 0, \quad \text{and} \quad r_{2\tau}^{h\rho*} = R\frac{1-\lambda r_{1\tau}^{\rho*}}{1-\lambda},\ \text{for } \tau \in \{A, B\};$$

in addition, investors run bank-τ after observing bad news on that bank ($s_\tau = \phi$) iff

$$\phi < \underline{\phi} \equiv \frac{(1-\lambda)\,\tilde{c}}{e^{\theta_T - \theta_M} R\,(2-\lambda\tilde{c})}$$

with $0 < \underline{\phi} < 1$*, but investors will not run the other bank.*

(ii) If investors are uncertainty averse, there are two equilibria: the risky equilibrium described in part (i), and a safe equilibrium where banks hold only the risk-free asset and the deposit contract is a safe deposit contract: $r_{1\tau}^{\sigma *} = r_{2\tau}^{l\sigma *} = 1$ *and* $r_{2\tau}^{h\sigma *} = 0$ *for* $\tau \in \{A, B\}$*. Furthermore, in a risky equilibrium, investors will run both banks after observing bad news on either of the two banks, that is,* $s_\tau = \phi$ *or* $s_{\tau' \neq \tau} = \phi$*, iff* $\phi < \underline{\phi}^2$*. There are no runs in the safe equilibrium.*

Theorem 3 characterizes the impact of uncertainty aversion on ex-ante optimal deposit contracts, bank runs, and systemic risk.[23] It shows that investor uncertainty aversion has two effects on banks runs. First, as discussed in Lemma 4, the presence of uncertainty aversion creates the possibility of systemic runs, even in cases where such runs would not occur under SEU. Thus, the presence of uncertainty aversion provides a channel for contagion and, thus, for systemic risk. However, Theorem 3 shows that the threshold level for bad news that triggers a run is lower when investors are uncertainty averse than when they are SEU investors, because $\phi < \underline{\phi}^2$. This means that uncertainty-averse investors are *slower* to run after observing bad news on a bank than SEU investors. This happens because, in a risky equilibrium, uncertainty-averse investors value more their deposit in a bank if they hold a deposit in the other bank as well. This means that, all else equal, an uncertainty-averse investor is more reluctant to run a bank after observing bad news on that bank. However, if the bad news is sufficiently bad to induce a run, the run spreads to the other bank. Thus, uncertainty-averse investors are less prone to bank runs, but when they run they generate a systemic run.[24]

[23] Note that in the optimal contract in the "risky" equilibrium, banks provide (partial) insurance against the liquidity shock, since the marginal utility of early consumption (measured by ψ) is sufficiently large. Insurance is limited (late investors strictly prefer not mimicking early investors) because $\tilde{c}$ is not too large.

[24] Theorem 3 depends on the assumption that utility is piecewise affine, as in (22). Affine utility guarantees that banks set the intermediate cashflow at the kink, so $r_{1\tau} = \frac{1}{2}\tilde{c}$. Thus, the optimal contract does not change when investors anticipate learning news. If u were strictly concave, results are similar but banks would decrease $r_{1\tau}$, unless there is an Inada condition for u. Because sufficiently bad news induces a run on both banks, it would be possible for early households to receive 0, so banks would write contracts that induce investors to set $S_a \geq 0$. Also, banks would have to decide if they were going to avert a fundamental run, or to allow a fundamental run (optimally choosing the contract with the risk of a run in mind). In either scenario, banks decrease the insurance provided to early type, $r_{1\tau}$.

3 Bank runs and the stock market

In the previous sections, we discussed the effect of uncertainty aversion on systemic risk within the banking sector. An important question is the potential connection between bank runs and the performance of other parts of the financial system such as the stock market. For example, in the recent financial crisis the near collapse of the (shadow) banking system was also associated with a substantial drop of the stock market. This observation raises the question of the transmission mechanism between the banking sector and the "real" sector. In this section, we show the contagion that we described in the previous section can spread beyond the banking sector and spill over to the stock market as well.

We modify our basic model as follows. We let bank A remain a bank, which now represents the overall banking sector, but bank B is now a stock company (or a mutual fund), denoted as firm B, which now represents the stock market. In this new interpretation, the stock company has access to type-B assets. In the spirit of Jacklin (1987), we posit that firm B promises to pay investors a dividend Δ_{1B} at time $t = 1$, and holds a portfolio $\{\sigma_{2B}, \rho_{2B}\}$ of the safe asset and type-B asset, until $t = 2$. Similar to our discussion in the previous section, bank A offers contract $r_A = \left\{r_{1A}, r_{2A}^l, r_{2A}^h\right\}$. Investors still face the possibility of a liquidity shock, so they would like to have insurance against it. For tractability, we will assume again that early investors have affine utility as in (22).

Lemma 5 *The stock company implements incentive-compatible cash flow* $\left\{r_{1B}, r_{2B}^l, r_{2B}^h\right\}$ *by setting* $\Delta_{1B} = \lambda r_{1B}$, $\sigma_{2B} = (1-\lambda)\, r_{2B}^l$, *and* $\rho_{2B} = (1-\lambda)\,\frac{r_{2B}^h}{R}$. *Late investors use the dividend to buy shares from the late consumers for price* $P_{1B} = (1-\lambda) r_{1B}$.

Lemma 5 follows directly from the reasoning described in Jacklin (1987). The stock company can replicate the payouts of a bank by committing to pay investors a certain dividend at $t = 1$. Early investors, because they must consume at $t = 1$, finance consumption using the dividend plus the proceeds from the sale of firm-B shares to late investors. Late investors, in turn, use the dividend they receive from firm B to purchase shares from selling early investors, and then consume at $t = 2$ the liquidating dividend they receive from firm B. Investors' portfolio allocation between banks and the stock market is as follows.

Lemma 6 *Each investor deposits half of their wealth in the bank and buys equity with the other half. If investors are uncertainty neutral, the risky equilibrium will be implemented. If investors are uncertainty averse, there are both the safe equilibrium and the risky equilibrium.*

Lemma 6 shows that the equilibrium from Theorem 3 is not sensitive to the institutional structure. In the spirit of Jacklin (1987), if no bad news arrives, the equilibrium allocation is identical whether the intermediaries are stock companies or banks. What happens if there is bad news?

Theorem 4 *Idiosyncratic risk leads to contagion between the banking sector and the stock market iff investors are uncertainty averse. That is, bad news about the bank harms the value of the stock, and bad news about the stock can produce a bank run, iff investors are uncertainty averse.*

Theorem 4 establishes a new mechanism for bad news to spread across segments of the financial sectors in an economy. Specifically, uncertainty aversion generates complementarity among different asset classes in the economy. Because of asset complementarity, bad news spreads directly across asset classes, due to investor preferences. This means that systemic risk extends to the broader financial sector, generating fragility for the whole financial sector.

Theorem 4 implies that a run on the banking sector is associated with negative performance of the stock market and leads to a "market crash." Our model also implies that investors would run to redeem their shares in mutual funds that have demandable features, such as money market funds, leading to a "breaking of the buck." Also, our model proposes a new channel through which financial crises spread from the banking sector to the real sector. Note that this new channel is driven by the impact of a bank run on investors' beliefs, generating a negative effect on stock market valuations. Thus, our theory differs from the more traditional view that a crisis in the banking sector negatively affects banks' lending and, thus, the real sector and stock market valuations.

Theorem 4 also implies that sufficiently negative news on the stock markets, which leads to a stock market "crash," also induces a run on the banking system. The bank run is then followed by a subsequent rebalancing of the long term investors' portfolios with a reinvestment of their holdings in the safe asset. Thus, a bank run generates a "flight to quality."

4 Multiple banks

In this section, we examine a simple extension of our basic model by allowing the presence of multiple banks and assets, with multiple sources of uncertainty in the economy. We show that the main results of our paper readily extend to the more general setting.

We modify our basic model as follows. Similar to Section 1, the economy is now endowed with $N+1$ types of assets: N classes of risky assets, $\tau \in \mathcal{N} \equiv \{1,..,N\}$, and a safe asset. Specifically, making at $t=0$ an investment in risky asset $\tau \in \mathcal{N}$ generates at $t=2$ a random payoff in terms of the safe asset: a unit investment in type τ asset produces at $t=2$ a payoff of R with probability p_τ and a payoff of 0 with probability $1-p_\tau$. Similar to Section 1, risky assets have an early liquidation option at $t=1$, so that liquidation of a fraction γ of the risky asset generates at $t=1$ a payoff $\gamma\ell$ of the safe asset and, at $t=2$ a payoff of $(1-\gamma)R$ with probability p_τ and a payoff of 0 with probability $1-p_\tau$.

Different from Section 1, the economy is characterized by multiple sources of uncertainty, as follows. The success probability on risky asset $\tau \in \mathcal{N}$, p_τ, depends again on the value of a parameter θ_τ, and we set $p_\tau(\theta_\tau) = e^{\theta_\tau - \theta_M}$, with $\theta_\tau \in [\theta_L, \theta_H] \subseteq [\theta_m, \theta_M]$. Investors are again uncertain over the vector $\overrightarrow{\theta} = \{\theta_\tau\}_{\tau=1}^N$, and assess that $\overrightarrow{\theta} \in \mathcal{C} \subset [\theta_L, \theta_H]^N \subseteq [\theta_m, \theta_M]^N$. We assume again that, for all $\overrightarrow{\theta} \in \mathcal{C}$, we have that $\Sigma_{\tau=1}^N \theta_\tau = N\theta_T + \kappa$. Investors are uncertain on the value of κ as well, and assess that $\kappa \in \mathcal{K} \equiv [-A, A]$. We assume that $N\theta_L < N\theta_T - A$ and $N\theta_H > N\theta_T + A$. We can interpret κ as representing the aggregate state of the economy at $t=2$, and θ_τ as measuring the exposure of each asset τ to the state of the overall economy. In this spirit, we will denote the combination $\{\overrightarrow{\theta}, \kappa\}$ as the "state of the economy" at $t=2$.

Bank τ offers investors the contract $r_\tau \equiv \left\{r_{1\tau}, r_{2\tau}^l, r_{2\tau}^h\right\}$ per dollar deposited in the bank. By depositing d_τ in bank τ at $t=0$, an investor receives a lifetime utility equal to

$$U_0 = \lambda u(S_a + \Sigma_{\tau=1}^N r_{1\tau} d_\tau) + (1-\lambda) \min_{\overrightarrow{\theta}_1} U_1\left(\overrightarrow{\theta}_1, \kappa\right)$$

where

$$U_1\left(\overrightarrow{\theta}_1, \kappa\right) = S_a + \Sigma_{\tau=1}^N \left[r_{2\tau}^l + p_\tau(\theta_\tau) r_{2\tau}^h\right] d_\tau.$$

Investors' assessments are again endogenous, and depend on the composition of their overall portfolio. Specifically, investors' assessments at $t = 1$ on the state of the economy at $t = 2$ are the solution to the minimization problem

$$\{\overrightarrow{\theta}^a, \kappa^a\} = \arg \min_{\{\overrightarrow{\theta}, \kappa\} \in S} U_1 \left(\overrightarrow{\theta}, \kappa \right),$$

and are characterized in the following Lemma.

Lemma 7 *Uncertainty-averse investors fear the worst about the aggregate state of the economy, and set $\kappa^a = -A$. If banks offer contracts that have similar risky payoffs, uncertainty-averse investors have "interior" assessments on the exposure of each asset to the aggregate uncertainty in the economy, $\overrightarrow{\theta}$:*

$$\theta_\tau^a = \theta_T - \frac{A}{N} + \frac{1}{N} \sum_{\tau'=1}^{N} \ln r_{2\tau'}^h d_{\tau'} - \ln r_{2\tau}^h d_\tau, \text{ for } \tau, \tau' \in \mathcal{N}, \ \tau \neq \tau'.$$

To proceed further, we make the following regularity assumption:

$$e^{\frac{1}{N-1}(N\theta_T - A - \theta_H) - \theta_M} R < 1 < e^{\theta_T - \frac{A}{N} - \theta_M} R,$$

Similar to the two-bank case, the first inequality guarantees that it is a negative NPV project to invest in the risky asset if at least one of the other banks does not; the second inequality guarantees that it is a positive NPV project to invest in the risky asset, if all of the banks invests. The following theorem shows that the basic results of our paper extend to the case of multiple banks.

Theorem 5 *In the absence of uncertainty aversion, the only equilibrium is the risky equilibrium, and local shocks stay local. In the presence of uncertainty aversion, there are both the risky equilibrium (where all banks invest in risky assets) and the safe equilibrium (where no banks invest in the risky asset), and all runs will be systemic.*

5 Increased uncertainty and financial crises

In this section, we examine the impact of the "extent" of uncertainty on financial system fragility and contagion. We show that increasing uncertainty makes the financial system more fragile and more prone to contagion and, thus, more vulnerable to systemic risk. In addition, we show that when aggregate uncertainty is very high, only the safe equilibrium with a credit crunch exists.

We measure the extent of uncertainty by the size of investors' core belief set, as follows. Let $\alpha \equiv \theta_H - \theta_T$ characterize the level of uncertainty that investors have for each individual bank. Thus, we interpret the parameter α as measuring the extent of "individual-bank" uncertainty, with the parameter A measuring the extent of "aggregate" uncertainty.[25] In this paper we take as exogenous the factors that may induce time series variations of the parameter α. However, Epstein and Schneider (2010) suggest that such variations in uncertainty may be the product of learning by uncertainty-averse agents.

The impact of increasing relative and aggregate uncertainty on the financial system is characterized in the following.

Theorem 6 *Let $\alpha > A/N$; there are critical values (defined in the Appendix) for individual-bank uncertainty, $\{\alpha_R(N), \alpha_C\}$, with $\alpha_R(N)$ increasing in N, aggregate uncertainty, $\{A_1, A_2\}$, and the number of banks in the banking sector, N_C, such that:*

1. *For $A \leq A_1$: the risky equilibrium exists; there is no contagion for $\alpha \leq \alpha_R(N)$; contagion and systemic runs exist for $\alpha > \alpha_R(N)$. The safe equilibrium with a credit crunch exists only for $\alpha \geq \alpha_C$. In addition, $\alpha_R(N) \leq \alpha_C$ if and only if $N \leq N_C$.*
2. *For $A_1 < A \leq A_2$: the risky equilibrium exists; there is contagion and systemic runs. The safe equilibrium with a credit crunch exists only for $\alpha > \alpha_C$.*
3. *For $A > A_2$: there is only the safe equilibrium with a credit crunch.*

Theorem 6 shows that both "individual-bank" and "aggregate" uncertainty affect the possibility of runs and the nature of equilibria in the banking sector in the economy. When both individual-bank

[25] Note that, by construction (and symmetry across banks), individual-bank uncertainty must be at least equal to the "per-capita" aggregate uncertainty: $\alpha \geq A/N$.

and aggregate uncertainty is low, that is, for $\alpha \leq \alpha_R(N)$ and $A \leq A_1$ the only equilibrium is the risky equilibrium. In this case, fundamental runs are possible following bad news on a bank's future expected profitability, but runs remain local and do not create contagion. At higher levels of individual-bank uncertainty, that is, for $\alpha > \alpha_R(N)$, bad news from one bank can spread to the other bank, thus creating contagion and systemic risk. Safe equilibria are also possible at high levels of individual-bank uncertainty, $\alpha \geq \alpha_C$. For intermediate level of aggregate uncertainty, $A_1 < A \leq A_2$, the risky equilibrium still exists, but it is always exposed to the possibility of contagion and, thus, systemic runs. Finally, at very high levels of aggregate uncertainty, $A > A_2$, there is only the safe equilibrium with a credit crunch. In this case, the financial system retrenches itself in a "safety mode," whereby banks invest only in the safe asset.

Note that the critical threshold level $\alpha_R(N)$ is an increasing function of the number of banks that are active in the economy. This means that a larger banking sector (greater N) has two opposing effects on systemic risk. First, when aggregate uncertainty is low, $A \leq A_1$, an increase of N has the effect of raising the threshold level $\alpha_R(N)$ above which contagion can happen, reducing exposure to systemic risk. This effect is due to the positive externality among banks created by uncertainty aversion that we identified in this paper. This reduction of exposure to systemic risk has a positive impact on ex-ante investor welfare.

There is, however, a second effect that works in the opposite direction. This countervailing effect is precisely due to the fact that, in our model, idiosyncratic risk can generate contagion and, thus, result in a run on the whole banking system. Specifically, the presence of a greater number of banks in the economy has the effect of increasing the exposure of the economic system to a larger number of idiosyncratic shocks that can trigger a systemic run. Thus, an increase of the number of banks increases, all else equal, the likelihood of systemic risk, with a negative effect on ex-ante investor welfare. This means that the overall effect of an increase in the number of banks in the economy on systemic risk is not a foregone conclusion.

6 Empirical implications and public policy

In this section, we discuss some of the empirical implications of our paper. We then suggest more general, although tentative, implications of our paper for the recent public policy debate surrounding the management of financial crises.[26]

1. *Financial crises and contagion.* The main implication of our analysis is that financial crises can originate in one sector of the economy and then propagate through the banking system to other sectors and, possibly, the stock market. The mechanism that triggers and propagates financial crises in our model is the deterioration of the fundamentals (i.e., a negative shock) in one asset class that leads to worsening expectations on future returns in other asset classes. The key distinguishing feature of our model is that the initial negative shock can be idiosyncratic in nature, and still create contagion in otherwise unrelated asset classes. These are new and testable implications.

2. *Lending booms.* A key mechanism in our model is that uncertainty-averse investors are more optimistic about one asset class when they hold a larger portfolio position in another asset class. This implies that good news about one industry, like an increase in productivity of risky investment for that industry, R, will result not only in increased lending to that industry, but also increased lending to other industries as well. This property is a direct outcome of the externality across portfolio holdings created by uncertainty aversion.

3. Contagion channels. Our paper identifies a new channel for contagion across banks in the economy. Existing literature has focused on the structure of the interbank market as a key driver of contagion in the banking system (see, e.g., Allen and Gale, 2000). Our model implies that an important determinant of contagion across banks is provided by the structure of investor portfolios. This means that empirical tests of contagion between banks must also account for the pattern of investor deposit holdings. We believe that a better understanding of the network of portfolio holdings, while beyond the scope of our paper, is a fruitful avenue for future research.

It is helpful to note that empirical predictions (1) and (3) allows us also to differentiate our model from other models based on SEU, and will help to overcome the problem of the potential observational equivalence between models based on uncertainty aversion and those based on standard SEU (see, for example, the discussion in Maenhout, 2004, and Skiadas, 2003, among others).

[26] Thakor (2015b) provides a comprehensive survey of such debate.

For example, a model with SEU, *a-la* Goldstein and Pozner (2004), would deliver results similar to ours only if the idiosyncratic shock affecting one asset class is of such magnitude to have a quantitatively meaningful impact on the overall market price of risk. In this case, however, one must wonder whether such shocks are really "idiosyncratic" or, in fact, "systemic." Calibration exercises would be needed to shed further light on this matter, which we leave to future research.

We conclude this section with a more general discussion of the lessons we learn from our paper for public policy regarding bank bailout strategies, asset sales and, more generally, the management of financial crises. These considerations are more tentative in nature and, we believe, provide interesting areas for future research.

The role of regulation to curb systemic risk and promote financial stability has been the object of extensive discussion in recent academic and public policy debate. To implement effective stabilization polices and regulations, it is critical to understand the source of systemic risk and to assess the nature of bailout policies that must be implement by a central bank to prevent bank runs.

If investors are uncertainty averse, our paper shows that the central bank must worry about idiosyncratic shocks that affect individual banks, since these shocks can have systemic effects. In addition, the implementation of the bailout policy depends on the size of the shocks affecting the banking sector. For sufficiently small shocks, the central bank can avert a run by bailing out just the affected bank. If the shock is large enough, however, the central bank must also bail out unaffected (potentially solvent) banks to avoid a systemic crisis. In contrast, if investors are not uncertainty averse, the central bank only needs to bail out the affected bank. In addition, one of the basic results of our paper is that uncertainty harms stability and creates the possibility of systemic runs. The financial system is more fragile in times of greater uncertainty. In these cases, regulatory authorities may wish to release relevant information that reduces such uncertainty, thus increasing the "resilience" of the financial and banking system.

Similarly, our paper has implications on a central bank's choice in the event of financial crisis between interventions through bailouts or asset sales. Specifically, the central bank can either provide capital directly to the banks to fund their short-term liquidity needs (a bailout, discussed above), or it can buy a bank's risky assets and replace them with the safe asset (asset sales). The distinction is important because bailouts inject liquidity without changing a bank's balance sheet, while asset sales change the risk structure of the bank's portfolio. If investors are uncertainty averse, and the shock is large enough, our paper suggests that the optimal intervention policy involves asset sales. However, the central bank must purchase assets from the *unaffected* bank, not from the affected bank. Bad news to one bank effectively shifts the composition of investors' portfolios toward the other bank's holdings, so investors become more pessimistic about the unaffected bank's holdings. The central bank will be able to purchases these assets at distressed prices, which means that, ex post, the central bank will make large profits from these asset sales. If investors are uncertainty neutral, there is no place for asset sales. By extension, our model also suggests that the crisis will be harsher in countries that are not allowed to use asset repurchases, like Europe, than in countries that utilize asset repurchases, like the United States.

Negative idiosyncratic shocks at any one bank will have a negative effect on equity capitalization at other banks, triggering a widespread banking crisis. In other words, an idiosyncratic shock on one bank depresses its equity value, and the negative sentiment spreads to other banks which may now see distressed equity valuations. This may result in banks facing binding minimum equity requirements and may force banks to raise new equity at distressed prices. Thus, honoring minimum equity requirements would be very costly to banks.

Our paper has also implications on the nature of credit crunches and the difficulty of central banks to address them. If banks believe that other banks are not lending, they will find it optimal to not lend as well, generating a self-fulfilling credit crunch. This result is due to the effect of uncertainty aversion on probabilistic assessments, not because the banks are financially constrained. Thus, providing liquidity to banks will not be sufficient to induce them to start lending again.

The foregoing discussion implies that a critical issue for public policy, while facing the possibility of a financial crises, is to ascertain first the magnitude of investor ambiguity aversion and, second, to assess the extent of uncertainty that is present in the economy at that very point in time. The increasing body of empirical and experimental evidence on the relevance of ambiguity aversion as a driver of investor behavior (which we discussed in the introduction) is an avenue to address the first question. The second component, which we think is a key issue for policy making, is clearly more challenging. The assessment of the extent of uncertainty in the economy is critical because, in light of Theorem 6, it will affect the kind of polices that a central bank must follow to stabilize the banking sector and prevent systemic runs. It is quite difficult, of course, to generate clear empirical measures of uncertainty. Among those, a possibility is to focus on dispersions of forecasts, such as analyst forecasts, as in Anderson, Ghysels and Juergens (2009). Another possibility is to use the CBOE Volatility Index, a measure of the implied volatility of S&P 500 index options, or VIX, which is sometimes refereed to as the "fear factor," as in Williams (2015). Other measures are considered in Baker, Bloom, and Davis (2013), Brenner and Izhakian (2015), and Gallant, Jahan-Parvar, and Liu (2015). We think that generating sharp measures on uncertainty is a key area of future research.

7 Conclusion

In this paper, we propose a new theory of systemic risk based on uncertainty aversion. We show that uncertainty aversion creates complementarities among investors' asset holdings, a feature denoted as uncertainty hedging. Because of uncertainty hedging, bad news on an asset class may spread to other asset classes, generating systemic risk. In our model, a system-wide financial crisis is due to a deterioration of investors' sentiment on the overall economy. The key feature of our model is that this negative sentiment can be triggered by an idiosyncratic event, which creates a wave of pessimism that produces a systemic crisis. A second implication of uncertainty hedging is that banks may individually refrain from investing in risky assets even if, collectively, it would be beneficial to do so. In these situations, risky asset are valued by investors at distressed prices, and banks invest only in the safe assets, a feature that we describe as a credit crunch. Finally, we derive empirical and public policy implications of our model.

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A Appendix

Proof of Lemma 1. Let $x = \{x_A, x_B\}$ be a vector of indicator variables for success of type A and B assets: $x \in \{0,1\}^2$. If the probability of success is $p = \{p_A, p_B\}$ the probability of x is $p_A^{x_A} p_B^{x_B} (1-p_A)^{1-x_A} (1-p_B)^{1-x_B}$. Thus, the relative entropy of p w.r.t. q is

$$R(p|q) = \sum_{x\in\{0,1\}^2} p_A^{x_A} p_B^{x_B} (1-p_A)^{1-x_A} (1-p_B)^{1-x_B} \ln \frac{p_A^{x_A} p_B^{x_B} (1-p_A)^{1-x_A} (1-p_B)^{1-x_B}}{q_A^{x_A} q_B^{x_B} (1-q_A)^{1-x_A} (1-q_B)^{1-x_B}}.$$

Because the log of a product is the sum of the logs, and probabilities sum to one, we can express this as

$$R(p|q) = R(p_A|q_A) + R(p_B|q_B)$$

where $R(p_\tau|q_\tau) = p_\tau \ln \frac{p_\tau}{q_\tau} + (1-p_\tau)\ln\frac{1-p_\tau}{1-q_\tau}$. Because $\frac{\partial^2 R}{\partial p_\tau^2} = \frac{q_\tau}{p_\tau} + \frac{1-q_\tau}{1-p_\tau}$, $R(p_\tau|q_\tau)$ is strictly convex in p_τ. Thus, $R(p|q)$ is strictly convex in $p = \{p_A, p_B\}$. Also, $\lim_{p_\tau\to 0^+} R(p_\tau|q_\tau) = \ln\frac{1}{1-q_\tau}$ and $\lim_{p_\tau\to 1^-} R(p_\tau|q_\tau) = \ln\frac{1}{q_\tau}$. Define $\underline{\eta}(q) = \min_{\chi\in Q} \ln\frac{1}{\chi}$, where $Q = \{q_A, 1-q_A, q_B, 1-q_B\}$. Therefore, if $\eta < \underline{\eta}(q)$, M, as the lower level set of a strictly convex function, is strictly convex. Note that this result generalizes: Theorem 2.5.3 of Cover and Thomas (2006) shows that relative entropy is additively separable in independent variables, and their Theorem 2.7.2 shows that it is strictly convex. ■

Proof of Lemma 2. The worst-case scenario solves $\min U_1\left(\overrightarrow{\theta}\right)$ s.t. $\frac{1}{2}(\theta_A+\theta_B) = \theta_T$, where

$$U_1\left(\overrightarrow{\theta}\right) = S_a + \sum_{\tau\in\{A,B\}} \left[w_\tau r_{1\tau} + (1-w_\tau)\left(r_{2\tau}^l + e^{\theta_\tau-\theta_M} r_{2\tau}^h\right)\right] d_\tau.$$

Let ψ be the multiplier on the constraint. $\frac{\partial L}{\partial\theta_\tau} = -e^{\theta_\tau-\theta_M} r_{2\tau}^h d_\tau (1-w_\tau) + \frac{\psi}{2}$. Because U_1 is convex in θ, FOCs are sufficient for a minimum. Setting $\frac{\partial L}{\partial\theta_\tau}|_{\theta_\tau=\check{\theta}_\tau} = 0$, and substituting into $\frac{1}{2}(\theta_A+\theta_B) = \theta_T$, this implies

$$\check{\theta}_\tau^a = \theta_T + \frac{1}{2}\ln\frac{r_{2\tau'}^h d_{\tau'}(1-w_{\tau'})}{r_{2\tau}^h d_\tau (1-w_\tau)}.$$

Thus, if $\check{\theta}^a_\tau(\Pi) \in [\theta_L, \theta_H]$, $\theta^a = \check{\theta}^a$. If $\check{\theta}^a_\tau < \theta_L$, $\frac{\partial L}{\partial \theta_\tau} < 0$ for all $\theta \in [\theta_L, \theta_H]$, so $\theta^a = \theta_L$. If $\check{\theta}^a_\tau > \theta_H$, $\frac{\partial L}{\partial \theta_\tau} > 0$ for all $\theta \in [\theta_L, \theta_H]$, so $\theta^a = \theta_H$. Therefore, (9) corresponds to the worst-case scenario. ■

Outline of Proof of Theorem 1. In equilibrium, banks offer efficient contracts and investors invest all their wealth in the banks ($S_a = 0$). Because $p_\tau(\theta_T) R > 1$, banks invest the entire portfolio of risk-neutral late investors in risky assets, so $r^l_{2\tau} = 0$. Further, banks equalize marginal utilities across states – banks set $r_{1\tau}$ so that early investors receive $r_{1A}d_A + r_{1B}d_B = c^*_1$, where $u'(c^*_1) = p_\tau(\theta_\tau) R$. By (20), $2 < c^*_1 < \frac{2e^{\theta_T - \theta_M} R}{\lambda e^{\theta_T - \theta_M} R + (1-\lambda)}$, which (substituting into the budget constraint) guarantees that all IC constraints are lax, $U_1(\theta_T) > c^*_1$. WLOG, it is optimal for banks to offer symmetric contracts and investors to balance investment across banks: $r_{1\tau} = \frac{1}{2}c^*_1$ and $d_\tau = 1$ for $\tau \in \{A, B\}$. The complete proof is available in the Technical Appendix. ■

Outline of Proof of Theorem 2. In equilibrium, banks individually offer contracts that maximize investors' payoff, so it is WLOG optimal for investors to invest their entire wealth with banks, $S_a = 0$. However, there is the potential for a coordination failure across banks, resulting in the inefficient safe equilibrium.

Risky Equilibrium: If investors have a balanced portfolio with exposure to risky assets, $r^h_{2A}d_A = r^h_{2B}d_B > 0$, by Lemma 2, $\theta_\tau = \theta_T$, so $p_\tau(\theta_\tau) R > 1$, and banks will invest the entire portfolio of late investors in the risky asset: $r^l_{2\tau} = 0$. Banks equalize marginal utilities across states, setting $r_{1A}d_A + r_{1B}d_B = c^*_1$, where $u'(c^*_1) = p_\tau(\theta_T) R$. With uncertainty aversion, it is strictly optimal for banks to set risky investment so that investors have a balanced portfolio of risky assets: $r^h_{2A}d_A = r^h_{2B}d_B$. Thus, in the risky equilibrium, it is WLOG optimal for banks to offer symmetric contracts, $r^{\rho*}_{1\tau} = \frac{1}{2}c^*_1$, $r^{l\rho*}_{2A} = 0$ and $r^{h\rho*}_{2A} = r^{h\rho*}_{2B}$, and for investors to invest equally in the two banks: $d_\tau = 1$.

Safe Equilibrium: If the other bank does not invest in the risky asset, $r^h_{2\tau'} = 0$, investors will be very pessimistic about any investment by this bank: $\theta_\tau = \theta_L$ for all $r^h_{2\tau} > 0$. Because $p_\tau(\theta_L) R < 1$, such investment is value destroying, so banks set $r^h_{2\tau} = 0$. Therefore, if the other bank does not invest in the risky asset, this bank will not either. Because $u'(2) > 1$, banks would like to provide more insurance against the liquidity shock, but cannot due to the IC constraints (setting $r_{1\tau} > 1$ would result in all investors running at $t = 1$) so banks set $r_{1\tau} = r^l_{2\tau} = 1$. It is WLOG optimal for investors to invest equally in the two banks: $d_\tau = 1$.

The complete proof is available in the Technical Appendix. ■

Proof of Lemma 4. We focus on fundamental runs: late investors will run a bank only if it is optimal to withdraw when no one else runs.

Suppose investors are SEU: $p_A = p_B = e^{\theta_T - \theta_M}$. Following the shock to bank τ, the payoff from staying in both banks is $p_\tau \phi r^h_{2\tau} + p_{\tau'} r^h_{2\tau'}$. The payoff of running only bank τ is $r_{1\tau} + p_{\tau'} r^h_{2\tau'}$, while the payoff of running only bank τ', is $r_{1\tau'} + p_\tau \phi r^h_{2\tau}$. Finally, the payoff of running both banks is $r_{1\tau} + r_{1\tau'}$. Because the IC is lax and the contract is symmetric, $p_{\tau'} r^h_{2\tau'} > r_{1\tau'}$, so investors will not run bank τ'. Investors will run bank τ if $r_{1\tau} > \phi p_\tau(\theta_T) r^h_{2\tau}$.

Suppose instead that investors are MEU, and there is bad news about bank τ. If investors stay in both banks, they receive $\min_{\overrightarrow{\theta} \in C} \{p_\tau(\theta_\tau) \phi r^h_{2\tau} + p_{\tau'}(\theta_{\tau'}) r^h_{2\tau'}\}$. If one investor runs only bank τ, she receives payoff $r_{1\tau} + e^{\theta_L - \theta_H} r^h_{2\tau'}$, while if she runs only bank τ', she receives payoff $r_{1\tau'} + e^{\theta_L - \theta_H} \phi r^h_{2\tau}$. Because $\phi < 1$, it is worse to run only bank τ' than only bank τ. If the investor runs both banks, she receives $r_{1\tau} + r_{1\tau'}$. Because $e^{\theta_L - \theta_M} R < 1$, $r_{1\tau'} > 1$, and $r^h_{2\tau'} = R\frac{1-\lambda r_{1\tau'}}{1-\lambda}$, $r_{1\tau} + e^{\theta_L - \theta_H} r^h_{2\tau'} < r_{1\tau} + r_{1\tau'}$. Therefore, the investor will either run both banks or neither.

The shock can either result in corner or interior beliefs. If the shock is so bad that it results in corner beliefs, investors run both banks.[27] Less severe shocks, $\phi > e^{-2(\theta_T - \theta_L)}$, result in interior beliefs. Staying in both banks provides investors with the payoff (applying Lemma 2 and symmetry) $2e^{\theta_T - \theta_H} \phi^{\frac{1}{2}} r^h_{2\tau}$. If the investor runs both banks, they receive payoff $2r_{1\tau}$. Thus, uncertainty-averse investors run both banks iff $r_{1\tau} > \phi^{\frac{1}{2}} p_\tau(\theta_T) r^h_{2\tau}$. ■

Outline of Proof of Theorem 3. Contracts are similar to those in Theorems 1 and 2, so the proof follows by similar logic. The banks offer contracts to investors, who optimally allocate resources across banks. The cutoff for runs follows by substituting the equilibrium contracts into the expression from Lemma 4. The complete proof is available in the Technical Appendix. ■

[27] From Lemma 2, the shock is severe enough to induce corner beliefs iff $\phi \leq e^{-2(\theta_T - \theta_L)}$; the payoff to staying in both banks is $e^{\theta_L - \theta_M} r^h_{2\tau}\left(e^{\theta_H - \theta_L}\phi + 1\right)$. On this region, $e^{\theta_H - \theta_L}\phi \leq 1$, so the payoff of staying in both banks is less than $2e^{\theta_L - \theta_M} r^h_{2\tau}$, which is strictly less than $2r_{1\tau}$.

Proof of Lemma 5. The stock company agrees to pay dividend Δ_{1B} at $t=1$, and hold portfolio $\{\sigma_{2B}, \rho_{2B}\}$, which are risk-free and type-B assets respectively, until $t=2$. The stock is traded at $t=1$ for price P_{1B}. Early investors are willing to sell their share of the stock for any $P_{1B}>0$, because they place no value on $t=2$ consumption. Thus, early investors receive, per share, Δ_{1B} from the stock company and P_{1B} from the late investors at $t=1$. Late investors are willing to buy the shares from the early investors iff it improves their utility. If investors are SEU, they value shares of the stock company at $\sigma_{2B}+e^{\theta_T-\theta_M}R\rho_{2B}$ per share, so they are willing to buy iff $\sigma_{2B}+e^{\theta_T-\theta_M}R\rho_{2B}>P_{1B}$. Because they invested all their funds at $t=0$, late investors can only reinvest the dividend, so market clearing requires that $\lambda P_{1B} \leq (1-\lambda)\Delta_{1B}$. If Δ_{1B} is not too large, this binds, so $P_{1B}=\frac{1-\lambda}{\lambda}\Delta_{1B}$. Thus, the early type receives $\Delta_{1B}+P_{1B}=\frac{1}{\lambda}\Delta_{1B}$ per share, while the late type receives $\frac{1}{1-\lambda}\left[\sigma_{2B}+e^{\theta_T-\theta_M}R\rho_{2B}\right]$ per share. Thus, if $\sigma_{2B}+e^{\theta_T-\theta_M}R\rho_{2B}\geq\frac{1-\lambda}{\lambda}\Delta_{1B}$, $P_{1B}=\frac{1-\lambda}{\lambda}\Delta_{1B}$.

Therefore, the stock company can implement the same cash flows as banking contract $\left\{r_{1B}, r_{2B}^l, r_{2B}^h\right\}$ by setting $\Delta_{1B}=\lambda r_{1B}$, $\sigma_{2B}=(1-\lambda)r_{2B}^l$, and $\rho_{2B}=(1-\lambda)\frac{r_{2B}^h}{R}$. r is incentive compatible, so $P_{1B}=(1-\lambda)r_{1B}$. The case with MEU investors follows with similar logic, except that they are even more willing to buy the shares, because different asset classes are complements (prices are the same because the stock is priced by cash in the market). ■

Proof of Lemma 6. Lemma 5 showed the stock company can implement $\left\{r_{1B}, r_{2B}^l, r_{2B}^h\right\}$ by promising dividend $\Delta_{1B}=\lambda r_{1B}$, holding risk-free assets $\sigma_{2B}=(1-\lambda)r_{2B}^l$, and type-$B$ assets $\rho_{2B}=(1-\lambda)\frac{r_{2B}^h}{R}$. By identical logic to Theorem 3, the optimal contract sets $r_{1\tau}=\frac{1}{2}\tilde{c}$, $r_{2\tau}^l=0$, and $r_{2\tau}^h=\frac{R}{1-\lambda}\left(1-\frac{\lambda}{2}\tilde{c}\right)$, and investors optimally invest \$1 in the bank and \$1 in the stock. ■

Proof of Theorem 4. Lemma 6 shows that the bank and stock company provide contracts which produce symmetric payoffs $r_{1\tau}=\frac{1}{2}\tilde{c}$, $r_{2\tau}^l=0$, and $r_{2\tau}^h=\frac{R}{1-\lambda}\left(1-\frac{\lambda}{2}\tilde{c}\right)$; the stock company provides this payoff by committing to dividend $\Delta_B=\frac{r_{1B}}{\lambda}$ and holding risky assets of $\rho_B=(1-\lambda)\frac{r_{2B}^h}{R}$. Suppose investors are uncertainty averse.

Consider first stock valuation. Late investors are willing to buy from early investors if $P_{1B}\leq e^{\theta_B^a-\theta_M}R\rho_{2B}$, where θ_B^a is from Lemma 2. Because $\tilde{c}<2\frac{e^{\theta_T-\theta_M}R}{\lambda e^{\theta_T-\theta_M}+(1-\lambda)R}$, $r_{1B}<e^{\theta_T-\theta_M}r_{2B}^h$: this constraint is lax in the absence of bad news ($\theta_B^a=\theta_T$). Thus, if there is no bad news, $P_{1B}=\frac{1-\lambda}{\lambda}\Delta_{1B}$. If there is a run on the bank, $\theta_B^a=\theta_L$, so $P_{1B}^{Run}=e^{\theta_L-\theta_M}R\rho_{2B}$, because $r_{1B}>1>e^{\theta_L-\theta_M}r_{2B}^h$. Because $P_{1B}^{Run}<P_{1B}$, a bank run harms stock market valuation. If there is bad news about the bank, but not strong enough to induce a run, $e^{\theta_B^a-\theta_M}r_{2B}^h=e^{\theta_T-\theta_M}\phi^{\frac{1}{2}}r_{2B}^h$, so the stock is harmed by bad news to the bank iff $\phi<\underline{\phi}^2$, where $\underline{\phi}=\frac{r_{1\tau}}{e^{\theta_T-\theta_M}r_{2\tau}^h}$.

It is optimal to run the bank iff $r_{1A}+e^{\theta_L-\theta_M}r_{2B}\geq 2e^{\theta_T-\theta_M}\phi^{\frac{1}{2}}\left[r_{2A}^h r_{2B}^h\right]^{\frac{1}{2}}$. By symmetry, this holds iff $\phi^{\frac{1}{2}}\leq\frac{1}{2}\left[\underline{\phi}+e^{\theta_L-\theta_T}\right]$. Because $e^{\theta_L-\theta_M}R<1$ and $r_{2\tau}^h<R$, $e^{\theta_L-\theta_T}<\underline{\phi}$. This is a strictly smaller cutoff for ϕ, so it is possible to have bad news about the bank that harms stock valuation without triggering a run.

Following bad news about the stock, by Lemma 2, if late investors stay in the bank, they receive utility $2e^{\theta_T-\theta_M}\phi^{\frac{1}{2}}\left[r_{2A}^h r_{2B}^h\right]^{\frac{1}{2}}$. Running the bank provides an investor with utility $r_{1A}+e^{\theta_L-\theta_M}\phi r_{2B}^h$, so it is optimal to run iff $r_{1A}+e^{\theta_L-\theta_M}\phi r_{2B}^h\geq 2e^{\theta_T-\theta_M}\phi^{\frac{1}{2}}\left[r_{2A}^h r_{2B}^h\right]^{\frac{1}{2}}$. Applying symmetry and the quadratic formula, running is optimal iff $\phi<\underline{\underline{\phi}}\equiv\left[\frac{1-\sqrt{1-e^{-\alpha}\underline{\phi}}}{e^{-\alpha}}\right]^2$, where $\underline{\phi}=\frac{r_{1\tau}}{e^{\theta_T-\theta_M}r_{2\tau}^h}$ and $\alpha=\theta_H-\theta_T$. It can easily be shown that $\underline{\underline{\phi}}<\underline{\phi}^2$: sufficiently bad news about the stock causes a bank run.[28]

Finally, there is no contagion when investors are uncertainty neutral. Uncertainty-neutral investors assess $\theta_\tau=\theta_T$: they run the bank iff there is bad news on the bank with $\phi\leq\underline{\phi}\equiv\frac{(1-\lambda)\tilde{c}}{e^{\theta_T-\theta_M}R(2-\lambda\tilde{c})}$, not affecting the stock. Bad news on the stock will depress P_{1B} to $e^{\theta_T-\theta_M}\phi\rho_{2B}$ if ϕ is low enough, but will not affect the bank. ■

Proof of Lemma 7. Bank τ offers contract $\left\{r_{1\tau}, r_{2\tau}^l, r_{2\tau}^h\right\}$ and investors invest d_τ in each bank. Uncertainty only affects the risky portion of the portfolio, so investors' worst-case scenario solves $\min_{\vec{\theta}\in C}\sum_{\tau=1}^N e^{\theta_T-\theta_M}r_{2\tau}^h d_\tau$ subject to $\theta_\tau\in[\theta_L,\theta_H]$ and $\sum_{\tau=1}^N\theta_\tau=N\theta_T+\kappa$, where $\kappa\in[-A,A]$. Because $N\theta_L<N\theta_T-A$, $\exists\tau$ s.t. $\theta_\tau>\theta_L$. Because $r_{2\tau}^h\geq 0$ and $d_\tau\geq 0$, $\kappa^a=-A$. Let γ_U be the multiplier on the constraint that $\sum_{\tau=1}^N\theta_\tau=N\theta_T-A$, let $\gamma_{\tau L}$ and $\gamma_{\tau H}$ be the respective multipliers for $\theta_\tau\geq\theta_L$ and $\theta_\tau\leq\theta_H$, and let L be the Lagrangian. Thus,

[28] This proof assumed that the shock was not sufficiently bad to induce the stock to liquidate their risky asset position. If the stock company liquidates their risky assets, investors immediately run the bank, because $r_{1A}>1$.

$\frac{\partial L}{\partial \theta_\tau} = -e^{\theta_\tau - \theta_M} r_{2\tau}^h d_\tau + \gamma_U + \gamma_{\tau L} - \gamma_{\tau H}$. $\theta_\tau = \theta_H$ iff $r_{2\tau}^h d_\tau < \underline{D} \equiv e^{\theta_M - \theta_H}\gamma_U$, and $\theta_\tau = \theta_L$ iff $r_{2\tau}^h d_\tau > \bar{D} \equiv e^{\theta_M - \theta_L}\gamma_U$. For $\theta_\tau \in (\theta_L, \theta_H)$, $\theta_\tau = \theta_M + \ln \frac{\gamma_U}{r_{2\tau}^h d_\tau}$. Define $A_L = \left\{\tau : r_{2\tau}^h d_\tau \geq \bar{D}\right\}$, $A_H = \left\{\tau : r_{2\tau}^h d_\tau \leq \underline{D}\right\}$, and $A_I = \left\{\tau : r_{2\tau}^h d_\tau \in \left(\underline{D}, \bar{D}\right)\right\}$; $N_L = |A_L|$, $N_H = |A_H|$, and $N_I = |A_I|$. Because $\sum_{\tau=1}^N \theta_\tau = N\theta_T - A$,

$$\theta_\tau = \frac{1}{N_I}\left[N\theta_T - A - N_H\theta_H - N_L\theta_L\right] + \frac{1}{N_I}\sum_{\tau' \in A_I} \ln\left[r_{2\tau'}^h d_{\tau'}\right] - \ln\left[r_{2\tau}^h d_\tau\right].$$

If all assessments are interior, $N_I = N$ and $N_H = N_L = 0$, so $\theta_\tau = \theta_T - \frac{A}{N} + \frac{1}{N}\sum_{\tau'=1}^N \ln r_{2\tau'}^h d_{\tau'} - \ln r_{2\tau}^h d_\tau$. ■

Proof of Theorem 5. The proof follows by identical reasoning to Theorem 3. For uncertainty-neutral investors, $\theta_\tau = \theta_T$, so investment in risky assets is positive NPV because $e^{\theta_T - \theta_M} R > e^{\theta_T - \frac{A}{N} - \theta_M} R > 1$. It is WLOG optimal for investors to set $d_\tau = \frac{2}{N}$ and banks to set $r_{1\tau} = \frac{1}{2}\tilde{c}$.

For uncertainty-averse investors, if banks select the same risky payoff, $r_{2\tau}^{h\rho*}$, investors set $d_\tau = \frac{2}{N}$, so $\theta_\tau = \theta_T - \frac{A}{N}$ by Lemma 7. Because $e^{\theta_T - \frac{A}{N} - \theta_M} R > 1$, the risky equilibrium is an equilibrium. However, if all banks except one select the same risky payoff, $r_{2\tau}^{h\rho*}$, for $\tau \neq \tau'$, but $r_{2\tau'}^h = 0$, Lemma 7 implies $\theta_{\tau'} = \theta_H$ and $\theta_\tau = \frac{1}{N-1}\left(N\theta^e - A - \theta_H\right)$. Because $e^{\frac{1}{N-1}(N\theta^e - A - \theta_H) - \theta_M} R < 1$, the safe equilibrium is an equilibrium and runs spread. ■

Proof of Theorem 6. By Theorem 5, banks offer symmetric contracts and investors select $d_\tau = \frac{2}{N}$. If banks choose symmetric $r_{2\tau}^h > 0$, Lemma 7 implies $\theta_\tau = \theta_T - \frac{A}{N}$. Banks are willing to invest in risky assets only if $e^{\theta_T - \frac{A}{N} - \theta_M} R > 1$, or equivalently, $A < A_2 \equiv N\left(\theta_T - \theta_M + \ln R\right)$. If $A > A_2$, the only equilibrium is the credit crunch. If banks invest in the risky asset, from Theorem 3, they would like like to set $r_{1\tau}^{\rho*} = \frac{\tilde{c}}{2}$ and $r_{2\tau}^{h\rho*} = \frac{R}{1-\lambda}\left(1 - \lambda r_{1\tau}^{\rho*}\right)$. IC constraint (12) must be satisfied: $e^{\theta_T - \frac{A}{N} - \theta_M} r_{2\tau}^{h\rho*} \geq r_{1\tau}^{\rho*}$, which holds iff $A \leq A_1 \equiv N\left[\theta_T - \theta_M + \ln r_{2\tau}^{h\rho*} - \ln r_{1\tau}^{\rho*}\right]$. Thus, the IC is lax iff $A < A_1$, but the IC binds iff $A > A_1$, so $r_{1\tau} = \frac{e^{\theta_T - \frac{A}{N} - \theta_M} R}{1 - \lambda + \lambda e^{\theta_T - \frac{A}{N} - \theta_M} R} < \frac{\tilde{c}}{2}$. Because $\frac{r_{2\tau}^{h\rho*}}{r_{1\tau}^{\rho*}} < R$, $A_1 < A_2$.

Suppose there is bad news on a bank that induces a run on that bank, and that the IC constraint is lax. Contagion occurs if investors find it optimal to run the other banks: if $r_{1\tau}^{\rho*} > e^{\theta_\tau^- - \theta_M} r_{2\tau}^{h\rho*}$, where θ_τ^- is the belief on bank τ following bad news on one bank. If α is small, $\alpha \leq \frac{A}{N-2}$, investors have corner beliefs after they run one bank: $\theta_\tau^- = \theta_L = \theta_T - \alpha$, so contagion occurs iff $\alpha > \theta_T - \theta_M + \ln r_{2\tau}^{h\rho*} - \ln r_{1\tau}^{\rho*}$. If α is large enough, $\alpha > \frac{A}{N-2}$, investors have interior beliefs after they run a bank, so by Lemma 7, $\theta_\tau^- = \theta_T - \frac{A+\alpha}{N-1}$, so contagion occurs iff $A + \alpha > (N-1)\left[\theta_T - \theta_M + \ln r_{2\tau}^{h\rho*} - \ln r_{1\tau}^{\rho*}\right]$. Alternatively, if the IC constraint binds, $r_{1\tau} = e^{\theta_\tau - \theta_M} r_{2\tau}^h$, $\theta_\tau^- < \theta_\tau$ because $\alpha > \frac{A}{N}$, so $r_{1\tau} < e^{\theta_\tau^- - \theta_M} r_{2\tau}^h$, and contagion occurs for all $A > A_1$.

If $A < A_1$, let $N_0 = \frac{A}{K} + 2$, where $K = \theta_T - \theta_M + \ln r_{2\tau}^{h\rho*} - \ln r_{1\tau}^{\rho*}$. For $N \leq N_0$, define $\alpha_R(N) \equiv K$; for $N > N_0$, define $\alpha_R(N) = (N-1)K - A$. If $N < N_0$, $A > (N-2)K$. If $\alpha > \frac{A}{N-2}$, $\alpha + A > (N-1)K$, so contagion occurs for all $\alpha > \frac{A}{N-2}$ when $N < N_0$. If $\alpha < \frac{A}{N-2}$, contagion occurs iff $\alpha > K = \alpha_R(N)$. Alternatively, if $N > N_0$, $A < (N-2)K$. If $\alpha \leq \frac{A}{N-2}$, $\alpha < K$, so there is no contagion. If $\alpha > \frac{A}{N-2}$, there is contagion iff $A + \alpha > (N-1)K$, or equivalently, iff $\alpha > \alpha_R(N)$. Therefore, there is contagion iff $\alpha > \alpha_R(N)$. Because $\alpha_R(N_0) = K$, α_R is continuous in N_0. Also, α_R is not affected by N for $N \leq N_0$, yet α_R is increasing in N for $N > N_0$.

Finally, the credit crunch exists iff it is optimal for one bank to set $r_{2\tau}^h = 0$ when all the other banks set $r_{2\tau'}^h = 0$. If $r_{2\tau'}^h = 0$ for all $\tau' \neq \tau$, $\theta_\tau = \theta_T - \alpha$ for all $r_{2\tau}^h > 0$ (Lemma 7). This is negative NPV iff $e^{\theta_T - \alpha - \theta_M} R < 1$, or equivalently, iff $\alpha \geq \alpha_C \equiv \left(\theta_T - \theta_M + \ln R\right)$. Because $R > \frac{r_{2\tau}^{h\rho*}}{r_{1\tau}^{\rho*}}$, $\alpha_C > \alpha_R(N_0)$, so there there exists a unique $N_C > N_0$ such that $\alpha_C = \alpha_R(N_C)$. Note $N_C = \frac{\ln R - \ln \frac{r_{2\tau}^{h\rho*}}{r_{1\tau}}}{\theta_T - \theta_M + \ln \frac{r_{2\tau}^{h\rho*}}{r_{1\tau}}}$. ■

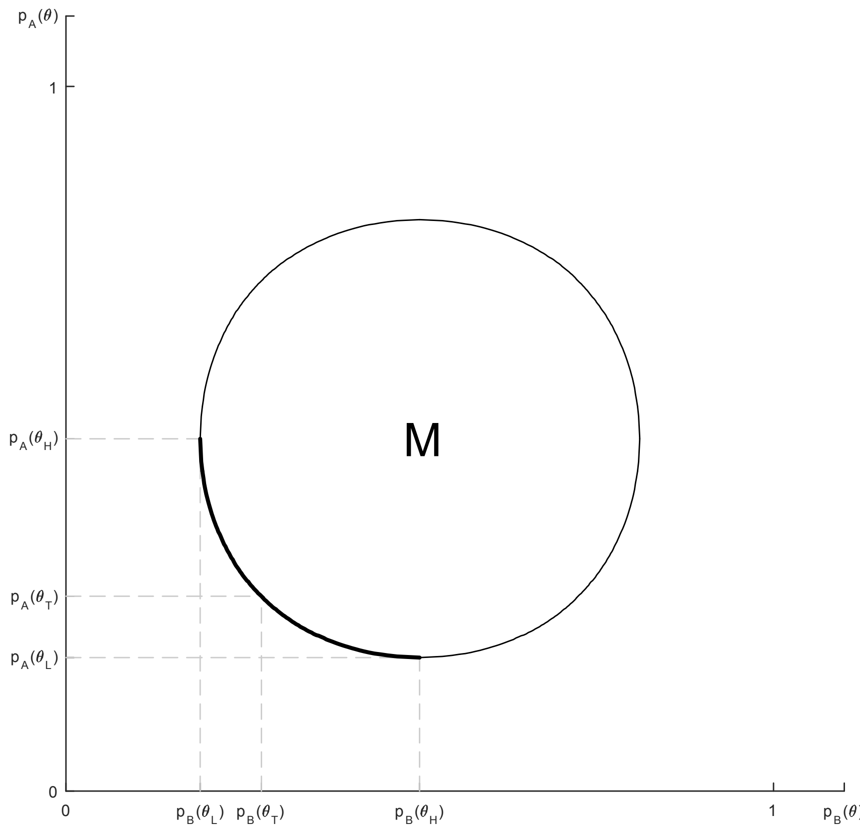

Figure 1: Core Belief Set: The figure represents the core belief set implied by the relative entropy criterion. The lower left boundary, which is darkened, represents the relevant portion of the core beliefs for investors with long positions in both risky assets.