**The Deterministic PDEs of Mathematical Finance**

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Major developments in mathematical finance have come from the study of two deterministic parabolic partial differential equations, the Nobel Prize winning Black-Scholes equation for stock options,

$$\frac{\partial u}{\partial t} = \frac{\sigma^2}{2} x^2 \frac{\partial^2 u}{\partial x^2} + rx \frac{\partial u}{\partial x} - ru,$$

and the Cox-Ingersoll-Ross equation for zero coupon bonds,

$$\frac{\partial u}{\partial t} = \frac{\sigma^2}{2} x \frac{\partial^2 u}{\partial x^2} + (\beta x + \gamma) \frac{\partial u}{\partial x} - xu,$$

where $(x, t) \in (0, \infty) \times [0, \infty)$. Each has a particular initial condition $u(x, 0) = u_0(x)$ of relevance in economics. In both models $\sigma$ is the volatility, $r$ is an interest rate, $\beta$ and $\gamma$ are also parameters given by the economic modeling.

We study these problems in weighted sup norm Banach spaces whose functions are unbounded near infinity (and possibly also near 0). The Black-Scholes equation is governed by a semigroup that is strongly continuous, quasicontractive, and chaotic. New extentions to time dependent coefficients will be given for this model.

The Cox-Ingersoll-Ross equation is governed by a strongly continuous quasicontractive semigroup, and the solution is given by a new type of Feynman-Kac formula. New extensions to more general potential terms will be explained as well as extensions to time dependent coefficients.